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## **A Unique and Understandable Way to Look at Three Phase Circuits**



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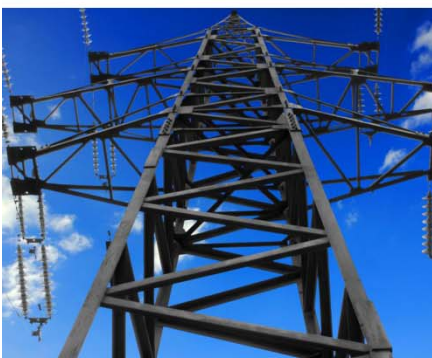
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**A Unique and Understandable Way to Look at  
Three Phase Circuits**

Robert J. Scoff, PE, PA, OH, & TN

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## **1. Introduction**

Single phase circuits have always been looked upon as relatively simple and easy to understand. This class will do a quick review of single phase circuits, reviewing some basic concepts from the 'AC Single Phase Energy' course, E-3001. Then, it will be shown that most multi-phase circuits can be shown to consist of a number of single phase circuits. One notable exception will also be analyzed, but in the end, there will be only one unknown worked with at a time, and thus only one equation at a time needs to be looked at. The whole idea is to make three phase circuits understandable to any person who has a basic understanding of vector algebra and trigonometry.

It is assumed that anyone taking this course has a basic understanding of vector algebra and trigonometry. To do any of the test questions, nothing more than a simple scientific calculator such as a TI-30 or TI-36 is necessary.

## **2. Definitions**

### **2.1 Units**

Following are definitions of various electrical terms that will be used as this course is developed.

#### **2.1.1 Voltage**

Electrical pressure. An analog would be hydraulic pressure, similar to the water pressure that exists in water pipes in a municipal water system. This pressure can exist without anything happening. Just look at a simple 9 volt battery. There's a voltage between the two terminals, but nothing happens until the battery is put in a flashlight, or other device, and the device is turned on.

#### **2.1.2 DC voltage**

This is a voltage that only has pressure in one direction. Batteries are examples of DC voltage sources.

#### **2.1.3 AC voltage**

This is a voltage that periodically changes direction. The most common AC voltage source is a sine wave source. That means that the voltage, or pressure, changes and goes positive and negative, just like a sine wave. The reason for sine wave voltage sources is that an AC voltage generator makes a sine wave of voltage because of its circular design. It is difficult to have it do anything else.

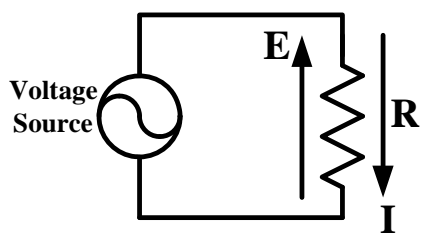
#### **2.1.4 Current**

Electrical flow. An analog would be hydraulic flow, similar to the water that flows when a spigot is turned on. There is a difference now. The water can just flow down the drain, and be gone. With electrical current, the current must return to the battery. The electrical switch can be

compared to the spigot, but the current just can't go down the drain, or into the air. It must return to the place where it came from, or the battery or source. The word source is often used when referring to something that has electrical voltage and supplies electrical current. In a hydraulic system, if the fluid is returned to a source, or recycled, it looks like an electrical circuit.

### 2.1.5 Ohm's law

Ohm's Law describes the effect of voltage on a resistance in an electrical circuit. This is perhaps the most powerful law in the electrical field, and, if understood, helps with the analyzing of most electrical problems. Ohm's Law states that  $E = I * R$ . Its hard to believe that such a simple equation could give beginning engineering students so much trouble.



$$E = I * R$$

Equation 2.1

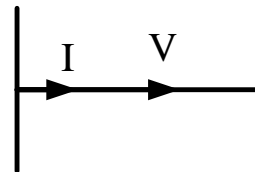


Figure 2.1A Vector Diagram Showing Current and Voltage In Phase

Figure 2.1 Simple Circuit Showing Voltage Across and Current Through a Resistor

Note that the voltage arrow points in one direction, which is toward the positive terminal of the source, and the current arrow points in the opposite direction, or from positive to negative

### 2.1.6 Power

Almost everything that works in the electrical field concerns power. The most basic definition of power is that  $P = E * I$ , where P is power, E is voltage and I is current. Other expressions for power are  $P = I^2 * R$  and  $P = E^2 / R$ . These can be gotten by using Ohm's Law ( $E = I * R$ ) and substituting the appropriate expression for E or I into the equation  $P = E * I$ . For example, if we start with  $P = E * I$ , and let  $E = I * R$ , and substitute that into  $P = E * I$ , P then equals  $I^2 * R$ .

$$P = E * I = I^2 * R = E^2 / R$$

Equation 2.2

### 2.1.7 Resistance

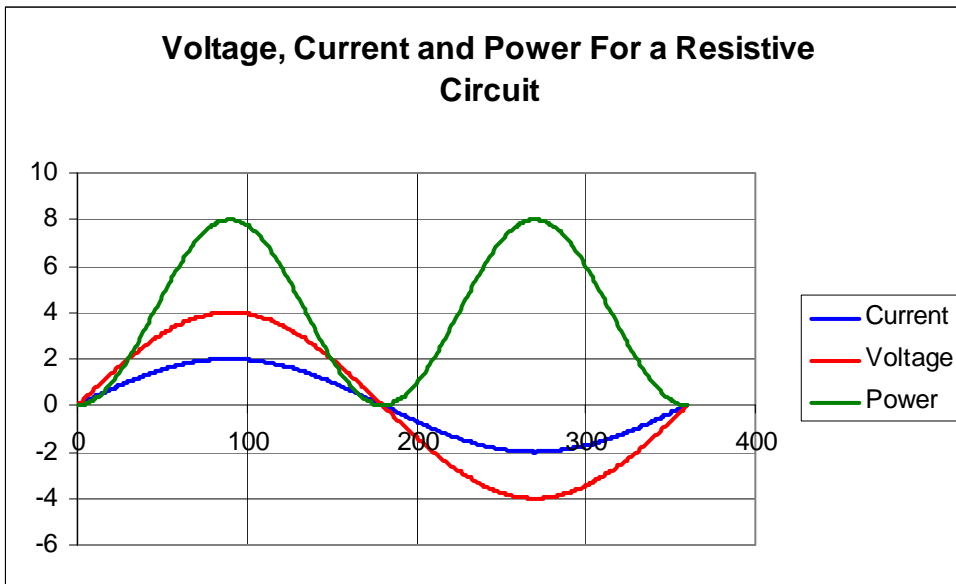
Resistance is the opposition to electrical current flow. When current flows through the resistor, there is a voltage drop across it and it dissipates power or gets hot. The resistor is the most common electrical circuit element. Just like its name implies, it opposes electrical current flow. To help understanding, and for simplicity, most of this course will emphasize resistance. Resistance works in both AC and DC circuits. Following is a graph showing voltage, current

and power in the simple one element circuit shown in Figure 2.1. For this example, maximum values are used and  $E = 4$  volts peak, and  $R = 2 \Omega$  (ohms). As this subject is developed, RMS (Root Mean Squared) will be used and the reason that most electrical power work uses RMS will be explained. It is particularly important to notice right now, that for a resistor, the current follows the voltage exactly, and the power is always positive. The power frequency is also double the frequency of the voltage and current. Examine the graph in figure 2.2 closely and see that this is so. This is true because  $P = E * I$  and when a sine wave is multiplied by another sine wave a double frequency sine wave results.

$$\sin^2 t = \frac{1}{2} - \frac{1}{2} \cos 2t$$

Notice that there is an offset that causes the power waveform to always be positive. This shows that a resistor always uses energy and never returns any energy to the source. Therefore, resistors get hot.

### Equation 2.3 The Result of Squaring a Sine Wave



**Figure 2.2 Graph of Instantaneous Voltage, Current, and Power for the circuit of Figure 2.1.**

### 2.1.8 Reactance

AC electrical circuits have another property called reactance. This is only true for AC circuits. Inductive and capacitive reactance will also be defined.

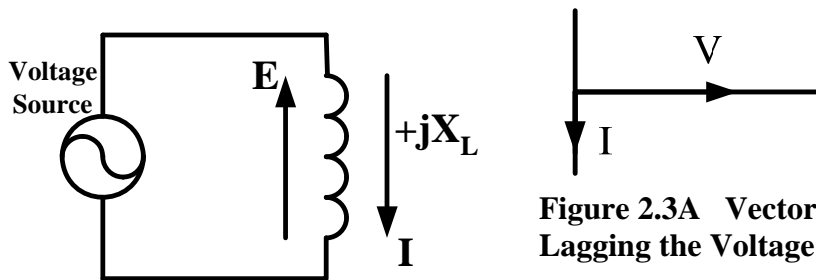
### 2.1.9 Inductive reactance

Inductive reactance is the opposition to AC current flow. An inductor consists of a wire wrapped around a core. Iron and iron alloys make the best cores, but air and non ferrous metal cores are also used. When an AC voltage is put across the inductor, current flows, but it lags voltage by 90 electrical degrees. A circuit showing a pure inductor across an AC voltage source

is shown in Figure 2.3. The voltage, current and power waveforms are shown in Figure 2.4. Note the similarity to the waveforms for the resistor, Figure 2.2. Also note that the average power is zero. This example still used 4 volts peak for the voltage and  $+j2 \Omega$  for the inductive reactance. The  $+j$  that is part of the reactance is very important. This causes the voltage and current to be 90 degrees out of phase.

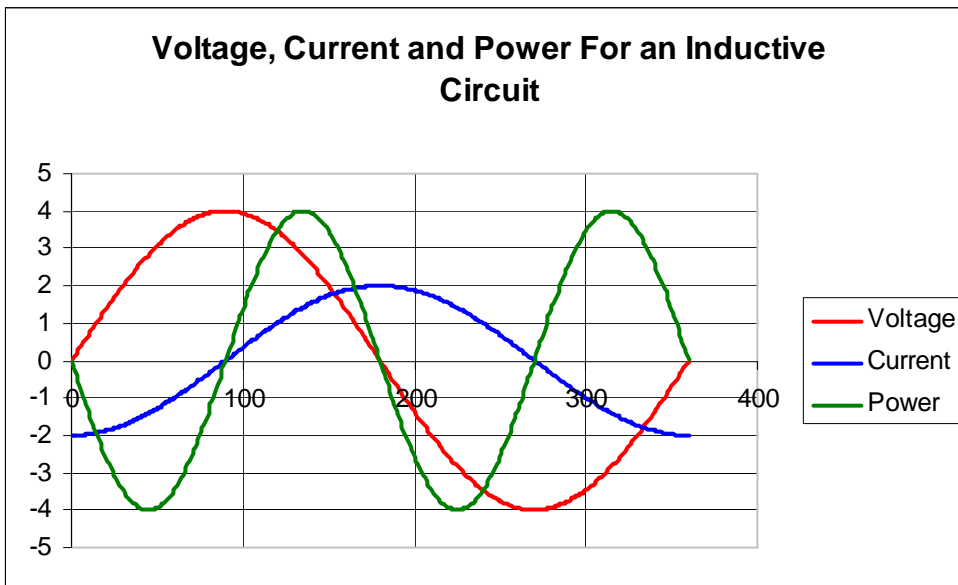
$X_L = +j2 \pi f L \Omega$  Note that this equation becomes  $X_L = +j377L \Omega$  for the typical US standard frequency of 60 cycles per second or hertz.

**Equation 2.4 Expression for determining  $X_L$**



**Figure 2.3A Vector Diagram Showing Current Lagging the Voltage by 90°**

**Figure 2.3 A Simple Circuit Showing the Voltage Across and the Current Through an Inductor**



**Figure 2.4 Graph of Instantaneous Voltage, Current, and Power for the circuit of Figure 2.3.**

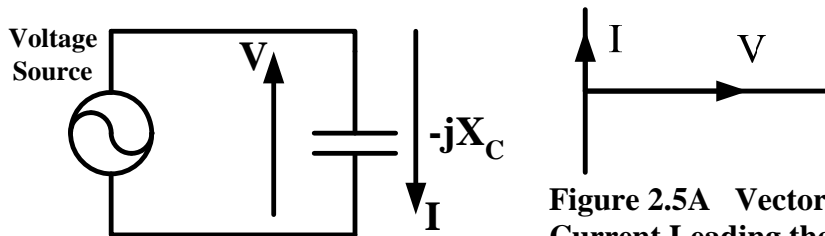
**2.1.10 Capacitive reactance**

Capacitive reactance is defined as the opposition to the flow of AC current flow in a capacitor. A capacitor is two metal plates separated by an insulator. When AC voltage is put across a

capacitor, current flows, but it leads the voltage by 90 electrical degrees. This is similar to what happens to an inductor, except the current leads the voltage, instead of lagging. Following is a diagram of a simple one element capacitive circuit (Figure 2.5). Note that there is no concern for the actual capacitance, just the capacitive reactance. Then figure 2.6 shows the voltage across, current through and power dissipated by the capacitor. The capacitive reactance is found by the following equation:

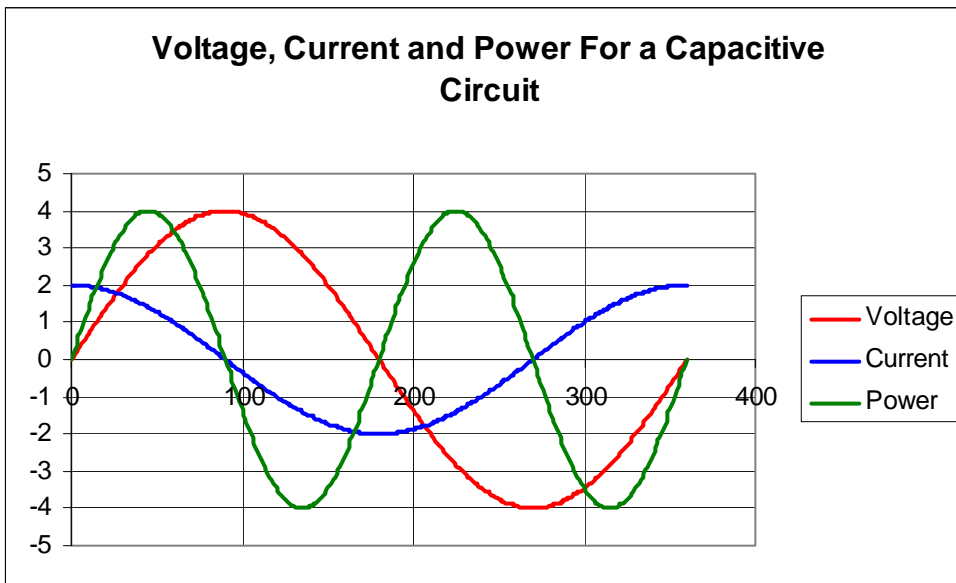
$$X_C = \frac{-j}{2 \pi f C} \Omega \quad \text{Note that this equation becomes } X_C = -j / 377C \Omega \text{ for the typical US standard of 60 cycles per second or hertz}$$

**Equation 2.5** Expression for determining  $X_C$



**Figure 2.5A** Vector Diagram Showing the Current Leading the Voltage by 90°

**Figure 2.5** A Simple Circuit Showing the Voltage Across and the Current Through a Capacitor



**Figure 2.6** Graph of Instantaneous Voltage, Current, and Power for the circuit of Figure 2.5

### 2.1.11 RMS

RMS stands for Root Mean Squared. The equation for determining the RMS value of a voltage or current is as follows:

$$\text{RMS} = \sqrt{\frac{\int_0^T [\text{V}_{\text{peak}} * f(t)]^2 dt}{T}} \text{ Volts}$$

**Equation 2.6 Expression for Determining the RMS Value of a Voltage Waveform**

If we want the RMS value of a current waveform, simply substitute  $I_{\text{peak}}$  for  $V_{\text{peak}}$ . Fortunately, it is not necessary to work out this integral. The only thing that is necessary is to know that if  $f(t)$  is a voltage sine wave, the RMS value is equal to:

$$\text{RMS} = \frac{V_{\text{peak}}}{\sqrt{2}} \text{ Volts}$$

For example, a 120 VAC, RMS voltage has a peak value of 170 volts. Look at Figure 2.2 and determine the RMS voltage and the RMS current. Note that the peak values are 4 volts peak and 2 amps peak.

**Equation 2.7 Equation for Determining the RMS Value of a Voltage Sine Wave**

The reason that RMS is used is that any RMS voltage that is placed across a resistor can be replaced by a DC voltage of the same value and the resistor will dissipate the same amount of energy. In other words, a 120 VAC, RMS voltage will make a resistor get just as hot as a 120 VDC battery placed across the same resistor.

You might, just for the fun of it, let  $f(t)$  be a sin wave {  $f(t) = \sin \omega t$  } and let the period  $T = 2 \pi$  seconds. That makes the frequency  $1/2\pi$  Hertz. Then  $\omega = 1$ . Then the integral is easy, because  $f(t) = \sin t$ , and the integral range is 0 to  $2\pi$ . Do the integration, try it, just for fun.

**2.1.12 Kirchoff's Laws**

No study of three phase circuits would be complete, or even possible, without looking at Kirchoff's Voltage and Current Laws.

**Kirchoff's Voltage Law states that the sum of the voltage rises around any closed loop is equal to the sum of the voltage drops.**

A simple example of this is Figure 2.1. It shows one voltage rise, the source, and one drop, the voltage across the resistor. The voltage of the source is the voltage across the resistor.

**Kirchoff's Current Law states that the sum of the currents entering a node (connection point) is equal to the sum of the currents leaving that node.**

Figure 2.7 shows a simple example, that will be used later in the course, of Kirchoff's Current Law. Note that we showed three currents going into the node, and none coming out of the node. The expression,  $I_1 + I_2 + I_3 = 0$ , is therefore true.

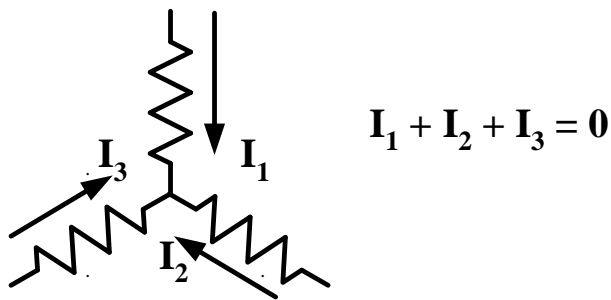


Figure 2.7 Kirchoff's Current Law Example

### 3. Three Phase Circuits

#### 3.1 Delta Delta Circuit

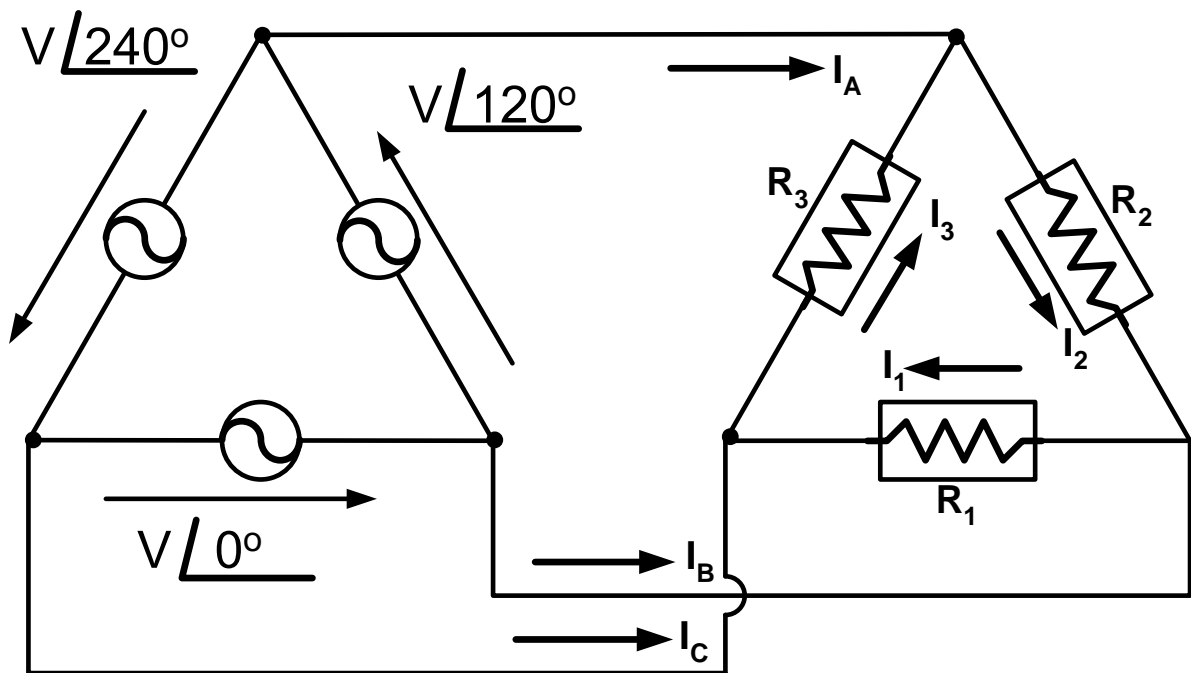
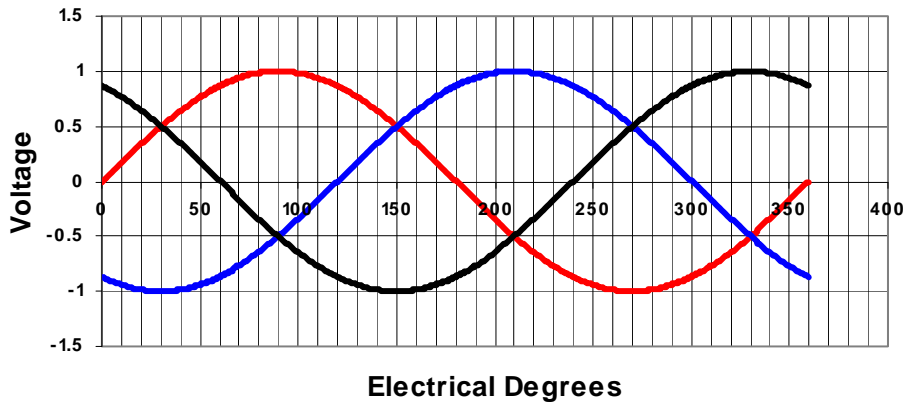


Figure 3.1 A Delta Delta Circuit Showing the Three Voltages, Each 120 Electrical Degrees Out of Phase with the Other Two Voltages, and Three Resistive Loads

Figure 3.1 shows a model of a typical three phase circuit with a delta source and a delta load. A good thing to do now is to look at the waveforms of the three voltages to show what the angles associated with the three voltages mean. Figure 3.2 shows that.

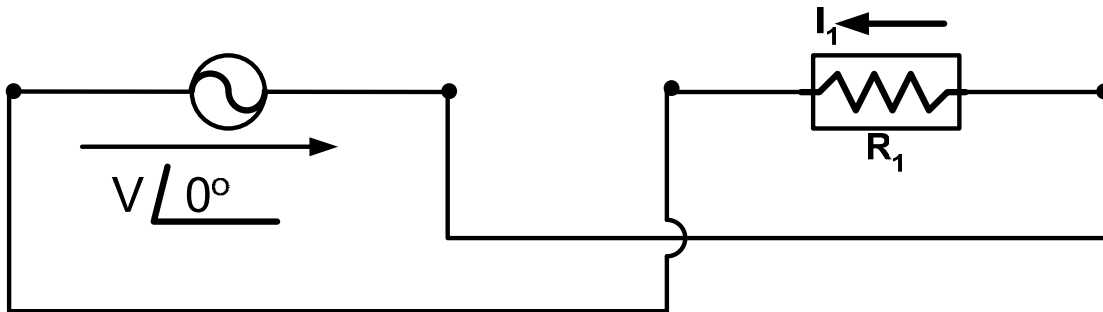
Note that the angle of the voltage is the angle, in degrees, where the waveform of the voltage goes through 0 volts in a positive direction. So the red waveform goes through 0 volts in a positive direction at 0 degrees, the blue waveform goes through 0 volts in a positive direction at 120 degrees and the black waveform goes through 0 volts in a positive direction at 240 degrees.

### Three Phase Power Waveforms



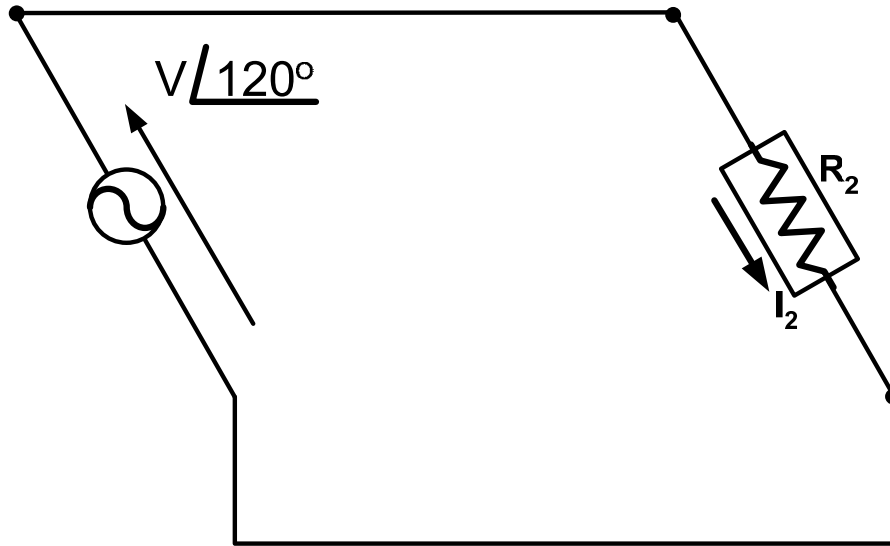
**Figure 3.2** Waveforms of the Three Voltages Shown in Figure 3.1. The Voltage Shown at 0 Degrees Is in Red, the Voltage Shown at 120 Degrees Is in Blue, and the Voltage Shown at 240 Degrees Is in Black.

The next step is to look at each of the three loads separately. An important thing to notice now is that the voltages in Figure 3.1 are drawn at the angle that was given them. In other words, the voltage that is said to be at 0 degrees is drawn at 0 degrees. The same is true of the other two voltages. Also, note that the load associated with each source is drawn at the same angle as the angle of the source. If this is kept clear, the solving for all unknowns is made easier. Figure 3.3 shows the 0 degree voltage and the load (in this case a resistor,  $R_1$ ). If  $V$  and  $R_1$  are known it is a simple matter to solve for  $I_1$ .  $I_1$  would be called the phase current for load  $R_1$ . Care needs to be exercised here to keep track of all angles in any calculations that are done.



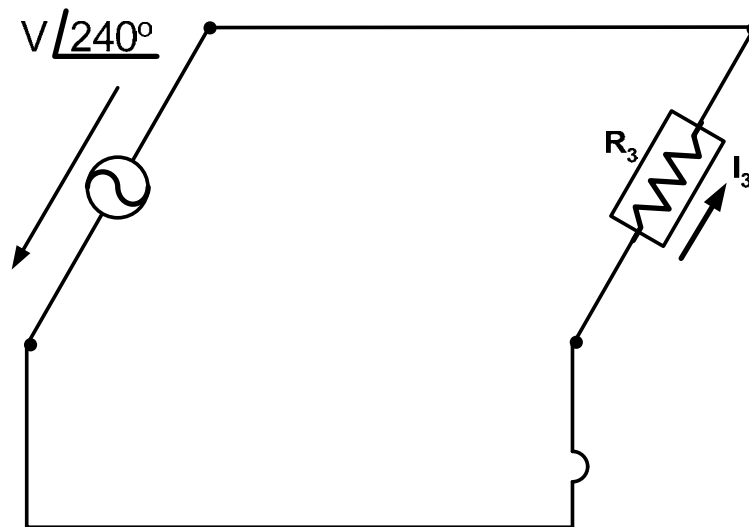
**Figure 3.3** A Single Phase Circuit Picked Out of the Three Phase Circuit of Figure 3.1

The next thing to look at is the single phase circuit associated with the voltage shown at 120 degrees. That voltage is across  $R_2$ , which is also drawn at 120°. Note that  $I_2$  is easily calculated if the voltage and resistance are known. However, the current,  $I_2$ , has an angle of 120 degrees associated with it. This needs to be kept track of. And again, this is called the phase current, because it is the current going through the phase resistor.



**Figure 3.4 The Second Single Phase Circuit Picked Put of the Three Phase Circuit of Figure 3.1.**

The third single phase circuit uses the voltage source that is at an angle of 240 electrical degrees. It is shown in figure 3.5. The resistor,  $R_3$ , is also drawn at 120 electrical degrees. Again, notice that the angle that the load is drawn at is the angle of the voltage source associated with it. Since purely resistive loads were chosen, the phase angles of the loads themselves are 0 degrees. However, the phase angles of the voltages and currents in those loads can be 0 degrees, 120 degrees, or 240 degrees.



**Figure 3.5 The Third Single Phase Circuit Picked Put of the Three Phase Circuit of Figure 3.1**

Now, all that has been done is that the three phase currents have been determined. When working with three phase circuits, the line currents are usually much more important. Look back at Figure 3.1 and notice that the line currents are called  $I_A$ ,  $I_B$ , and  $I_C$ . Each of the three nodes of the load has 2 currents entering it and one current leaving it. It is now easy to solve for the three

line currents by the use of Kirchoff's Current Law. That law says that whatever enters a node must instantaneously leave that node. Therefore:

$$\mathbf{I_A + I_3 = I_2}$$

$$\mathbf{I_C + I_1 = I_3}$$

$$\mathbf{I_B + I_2 = I_1}$$

### Equation 3.1 Node Equations for Figure 3.1

Solving for IA, IB, and IC gives the following:

$$\mathbf{I_A = I_2 - I_3}$$

$$\mathbf{I_B = I_1 - I_2}$$

$$\mathbf{I_C = I_3 - I_1}$$

### Equation 3.2 Equations for Line Currents

Let's do a simple example. Let the voltages be 480 volts and the three resistors be 480 ohms. Then:

$$\mathbf{I_1 = 1 \angle 0^\circ \text{ Amps}, I_2 = 1 \angle 120^\circ \text{ Amps}, I_3 = 1 \angle 240^\circ \text{ Amps}}$$

### Equation 3.3 Phase Currents for Simple Example

Solving for the line currents gives:

$$\mathbf{I_A = I_2 - I_3 = 1 \angle 120^\circ - 1 \angle 240^\circ = 1.732 \angle 90^\circ \text{ Amps}}$$

$$\mathbf{I_B = I_1 - I_2 = 1 \angle 0^\circ - 1 \angle 120^\circ = 1.732 \angle -30^\circ \text{ Amps}}$$

$$\mathbf{I_C = I_3 - I_1 = 1 \angle 240^\circ - 1 \angle 0^\circ = 1.732 \angle 210^\circ \text{ Amps}}$$

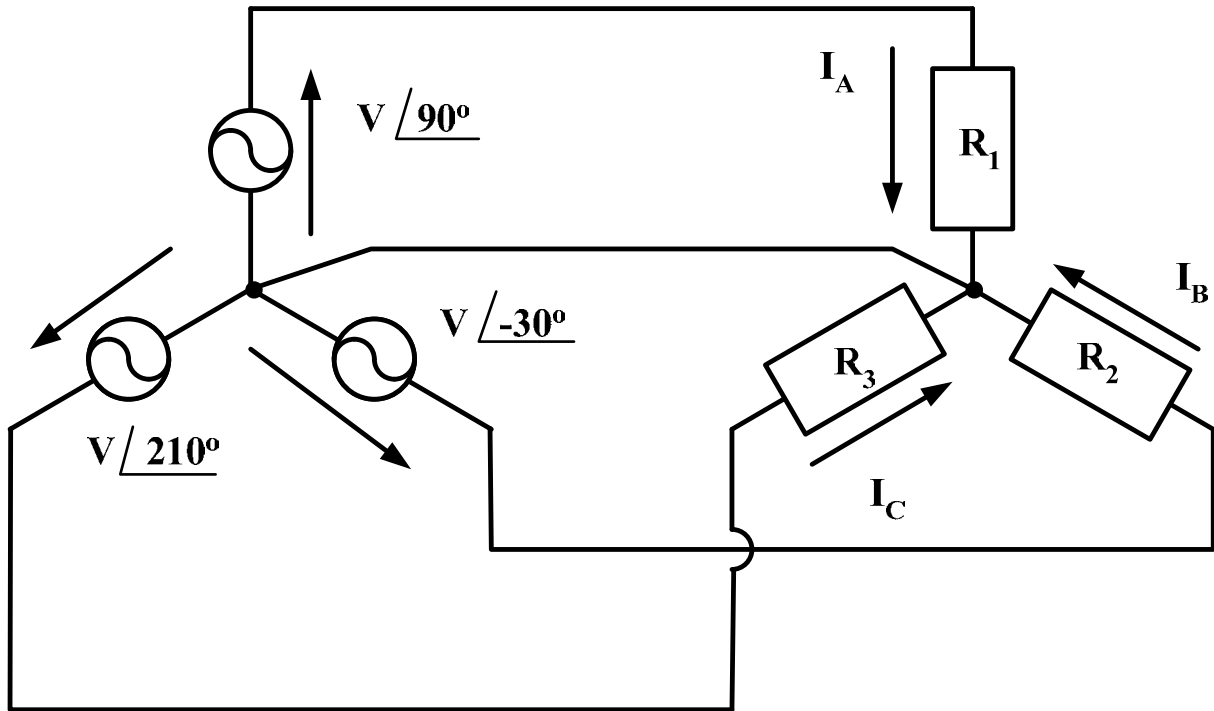
### Equation 3.4 Solution for Line Currents

An interesting thing to notice here is that the line currents are the square root of three amps and the phase currents are 1 amp. So, for this balanced three phase circuit, the line currents are the square root of three times the phase currents. This is something all the text books say, but this way of looking at the problem takes all the confusion and memorization out of it. Do notice, however, that the phase angles are kept track of, even though most measuring techniques do not

measure this item. The reason is that to get accurate magnitudes, the phase angles must be kept track of in the calculations. Otherwise, hopeless confusion and wrong results result.

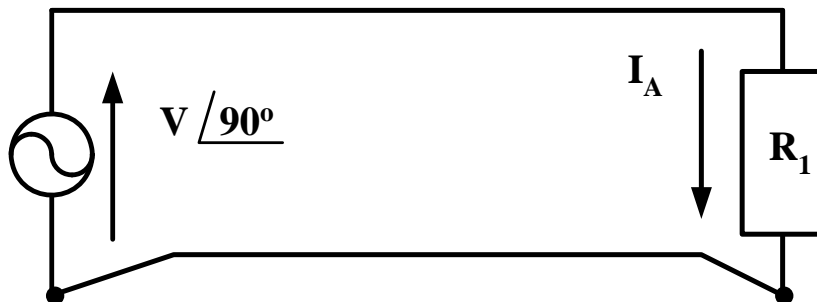
### 3.2 Wye Wye Circuits

The next common three phase circuit used in the world is the wye wye configuration. This is shown in figure 3.6.



**Figure 3.6 Wye Wye Three Phase Configuration**

Now, let's do the same thing that we did for the Delta Delta configuration. Let's find the three single phase circuits. They are shown in figures 3.7, 3.8, and 3.9.



**Figure 3.7 The First Single Phase Circuit In the Wye Wye Connection**

The first equation would be as follows:

$$I_A = \frac{V \angle 90^\circ}{R_1}$$

Equation 3.5 The First Wye Wye Equation

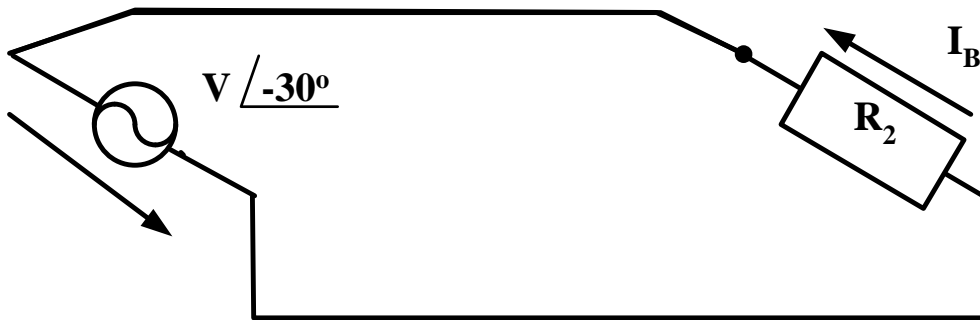


Figure 3.8 The Second Single Phase Circuit In the Wye Wye Connection

The second equation would be as follows:

$$I_B = \frac{V \angle -30^\circ}{R_2}$$

Equation 3.6 The Second Wye Wye Equation

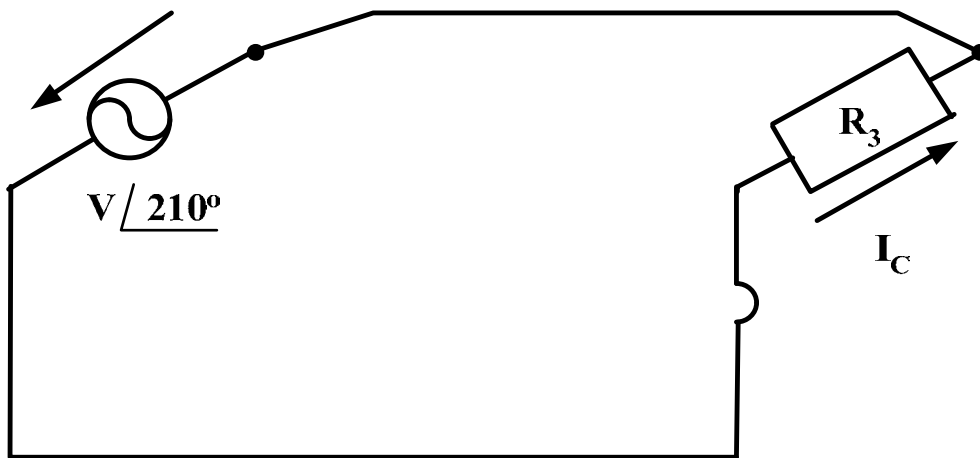


Figure 3.9 The Third Single Phase Circuit In the Wye Wye Connection

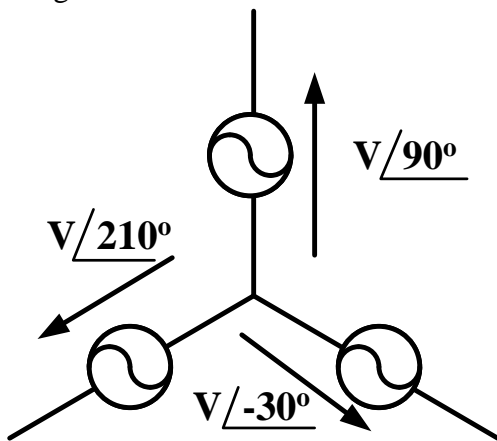
The third equation would be as follows:

$$I_C = \frac{V/210^\circ}{R_3}$$

**Equation 3.7 The Third Wye Wye Equation**

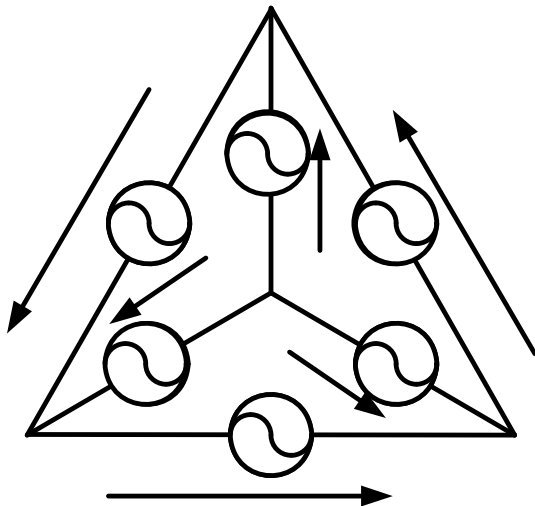
**3.3 Some General Observations and Wye Delta Source Transformations**

Some general observations are now in order. First notice that the line currents are the same as the phase currents. But the line voltages are not equal to the phase voltages. There is an easy way to determine the line voltages. Let's just look at the three phase Wye source. This is shown in figure 3.10.



**Figure 3.10 A Three Phase Wye Source**

If we just look at Figure 3.10 and put sources between each of the external end points of the Wye, we can determine the voltage that would exist from each of the points to any other point. As a matter of fact, we can make a make believe Delta as shown in Figure 3.11.



**Figure 3.11 An Illustration Showing How a Wye Source Can Be Seen as a Delta Source**

Now, if a Wye source is feeding a Delta load, the neutral of the Wye can be ignored, and all calculations can be done as if there were a Delta source. An easy thing to do is to infer the angles of the sources directly off of the drawing. The make believe Delta voltages could be determined by drawing the equilateral triangle, placing the Wye inside of it (as shown in Figure 3.10) and just accurately measuring the lengths. A more practical method is to just use the properties of the triangle and some trigonometry to see that the Delta Voltages are equal to the square root of three times the Wye voltages. And the angles that are used could come straight off of the picture. If a delta source is feeding a Wye load, there is no place to connect the neutral. If the load is balanced, this is no problem. The neutral point of the load is just assumed to be at zero volts. Referring to Figure 3.6, it is seen that there is a neutral wire connecting the center of the source to the center of the load. In the practical electrical world, this is what is usually done. As a matter of fact, for a real Wye source, this is a common way to connect lighting loads in industrial and commercial facilities. If there is a three phase 480 volt Wye source, the voltage from each leg to neutral is 277 volts. Hence the common 480/277 three phase four wire systems. The other common three phase four wire Wye source is 208 volts. This is the 208/120 volt system prevalent in commercial establishments.

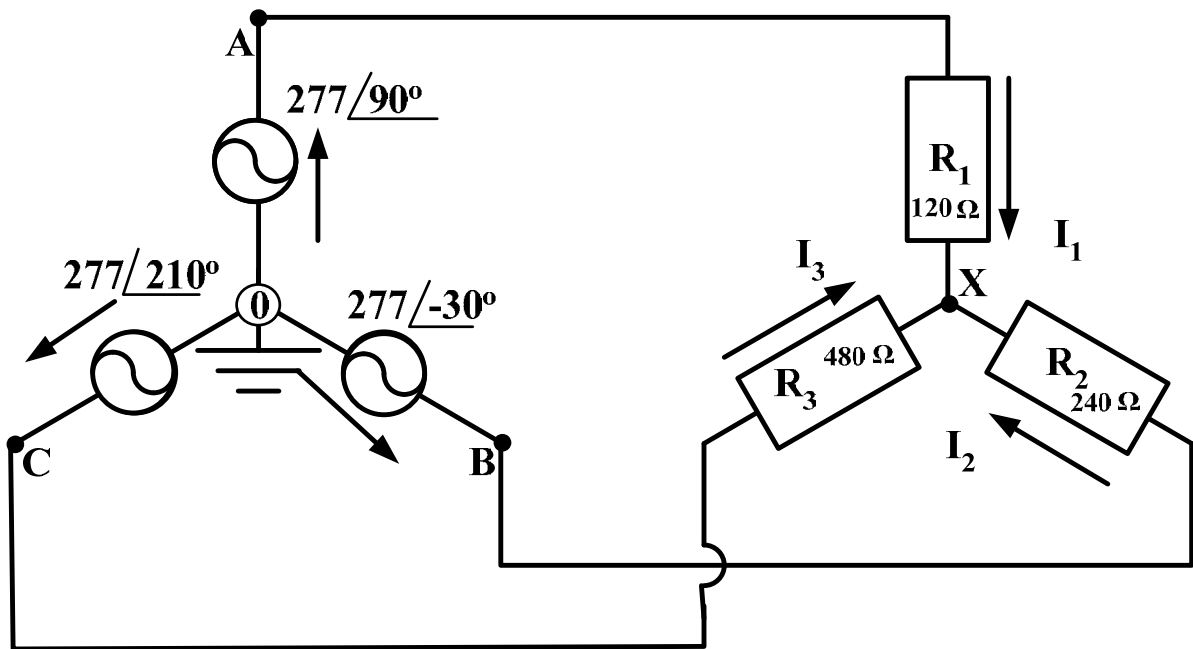
### 3.4 Unbalanced Loads

If the loads are all balanced, the phase currents are all equal and the line currents are all equal. This is true for any Wye Delta combination that is available. For unbalanced loads (loads where the three resistors are not equal) the above techniques can be used to determine the phase and line currents. Care has to be taken in keeping track of the phase angles. Mistakes made here in not keeping track of all of the details leads to errors.

The only unusual case is if a Delta source has an unbalanced Wye load. This is so uncommon that it is virtually unknown in the practical world. If it does occur, the voltage at the center of the Wye load must be determined. The easiest way to explain it is to give an example. Consider a Delta source, say 480 volts. Let the load be three resistors of 120  $\Omega$ , 240  $\Omega$ , and 480 $\Omega$ . The drawing would be as shown in Figure 3.11. Note that an imaginary Wye source is placed inside of the real Delta source. Now, the imaginary neutral of the imaginary Wye can be used as if it actually existed. The 480 volt Delta voltages at the angles shown are replaced by 277 volt Wye sources at the angles shown. It must again be emphasized that great care needs to be taken to keep track of the phase angles as well as the magnitudes of the voltage sources, calculated currents, and in the case of reactive loads, impedances.

#### Figure 3.11 A Delta Source Driving an Unbalanced Wye Load

Figure 3.11 looks complicated. But its not if we only look at what is important. First we decide what we want to know. How about  $I_1$ ,  $I_2$ , and  $I_3$ . If we only look at the original Delta source, solving for the currents is pretty complicated. Let's make believe that we have a Wye inside the Delta. It has a make believe neutral. Can we do that? Of course, after all we just made it up. Let's call the point in the center (with the circle) zero volts. That is now our reference. It doesn't exist, but so what. This drawing will now be simplified, removing the real Delta, and replacing it with the imaginary Wye. Figure 3.12 shows this.



**Figure 3.12 The Real Delta Source Completely Replaced By the Imaginary Wye Source**

Examination shows that there are 5 voltages on this drawing. They are the voltages at 0, A, B, C, and X. The only one that we don't know the value of is X. Since there is one unknown, there ought to be one equation. And there is, and it follows.:

$$\mathbf{I_1 + I_2 + I_3 = 0}$$

**Equation 3.8 The One Equation**

Then, we need to find expressions for I1, I2, and I3. And here they are.

$$\begin{aligned} \mathbf{I_1} &= (\mathbf{V_A} - \mathbf{V_X}) / \mathbf{R_1} = (277\angle 90^\circ - \mathbf{V_X}) / 120 \\ \mathbf{I_2} &= (\mathbf{V_B} - \mathbf{V_X}) / \mathbf{R_2} = (277\angle -30^\circ - \mathbf{V_X}) / 240 \\ \mathbf{I_3} &= (\mathbf{V_C} - \mathbf{V_X}) / \mathbf{R_3} = (277\angle 210^\circ - \mathbf{V_X}) / 480 \end{aligned}$$

**Equation 3.9 The Next Step**

Then putting it all together gives:

$$(277\angle 90^\circ - \mathbf{V_X}) / 120 + (277\angle -30^\circ - \mathbf{V_X}) / 240 + (277\angle 210^\circ - \mathbf{V_X}) / 480 = 0$$

**Equation 3.10 One Equation One Unknown**

Solving for the only unknown:

$$V_X = 104.15 \angle 70.79^\circ \text{ Volts} = 34.27 + j98.35 \text{ Volts}$$

### Equation 3.11 The Only Unknown Voltage

And then the three currents are found:

$$I_A = 1.52 \angle 100.86^\circ \text{ Amps} \quad I_B = 1.31 \angle -49.04^\circ \text{ Amps}$$
$$I_C = 0.75 \angle -139.18^\circ$$

### Equation 3.12 The Three Currents

Fortunately, the above process doesn't need to be done very often. Most of the time, three phase Delta circuits are only going to be concerned with Delta loads or with balanced Wye loads. With balanced Wye loads, the voltage at the node of the Wye load can be assumed to be 0. Then the three currents can be found by pretending the source is a Wye, and connecting the make believe 0 of the equivalent Wye source to the junction of the Wye load, and solving 3 single phase circuits.

## 3.5 Reactive Loads

To help understanding, I have only used resistive loads. The real world is not nearly so nice. Most loads are inductive or capacitive. The process described in this course works for any load. The load just needs to be described by its vector form. The phase angle associated with a resistor is, of course, 0 degrees. An inductive load has a positive angle of up to 90 degrees, and a capacitive load has a negative angle of up to -90 degrees. Since this work already works with phase angles and vector algebra, extending this work to reactive components is a straight forward, but not necessarily easy, process.

## 3.6 Phase Sequence

Finally, no study of three phase circuits is complete without talking about phase sequence. Look at Figure 3.12 again. Notice that the three terminals of the Wye source are labeled A, B, and C.  $R_1$  is connected to A,  $R_2$  is connected to B, and  $R_3$  is connected to C. We can call this ABC sequence for the load shown. Now, if terminal A is left where it is, and terminals B and C are reversed, the load has the opposite phase sequence. An interesting exercise now would to determine  $V_X$  for both phase sequences, and see if the voltages across the three resistors changes. My logic and intuition say that total power and the power of each resistor stay the same. I have worked this out, and the results are as follows:

$$V_X = 104.69 \underline{/109.11^\circ} \text{ Volts} = -34.27 + j98.92 \text{ Volts}$$

$$I_A = 1.51 \underline{/79.11^\circ} \text{ Amps}$$

$$I_B = 1.31 \underline{/ -130.89^\circ} \text{ Amps}$$

$$I_C = 0.76 \underline{/ -40.89^\circ} \text{ Amps}$$

Equation 3.13 Phase Currents for Figure 3.13

The power dissipated by each of the three resistors would remain the same for both phase sequences. That is because the magnitude of the currents through the three resistors remained the same for both phase sequences. Note that only two phase sequences are possible for three phase circuits. If the powers were calculated for the three resistors, the results would be:

$$P_A = I_A^2 * R_A = 273.61 \text{ Watts} \quad P_B = I_B^2 * R_B = 411.86 \text{ Watts}$$

$$P_C = I_C^2 * R_C = 277.25 \text{ Watts}$$

**This makes the total power equal to 962.72 Watts.**

Equation 3.14 Powers for the Unbalanced Wye Circuit

Notice that, even though, the phase angles changed the magnitudes of the currents stayed the same. And then, if we wanted to measure the voltage across the three resistors, they would also remain the same. Changing the phase sequence would not change the magnitude of the currents through or the voltages across the three resistors in the unbalanced load. This also works for reactive loads, but the math becomes a bit more confusing.

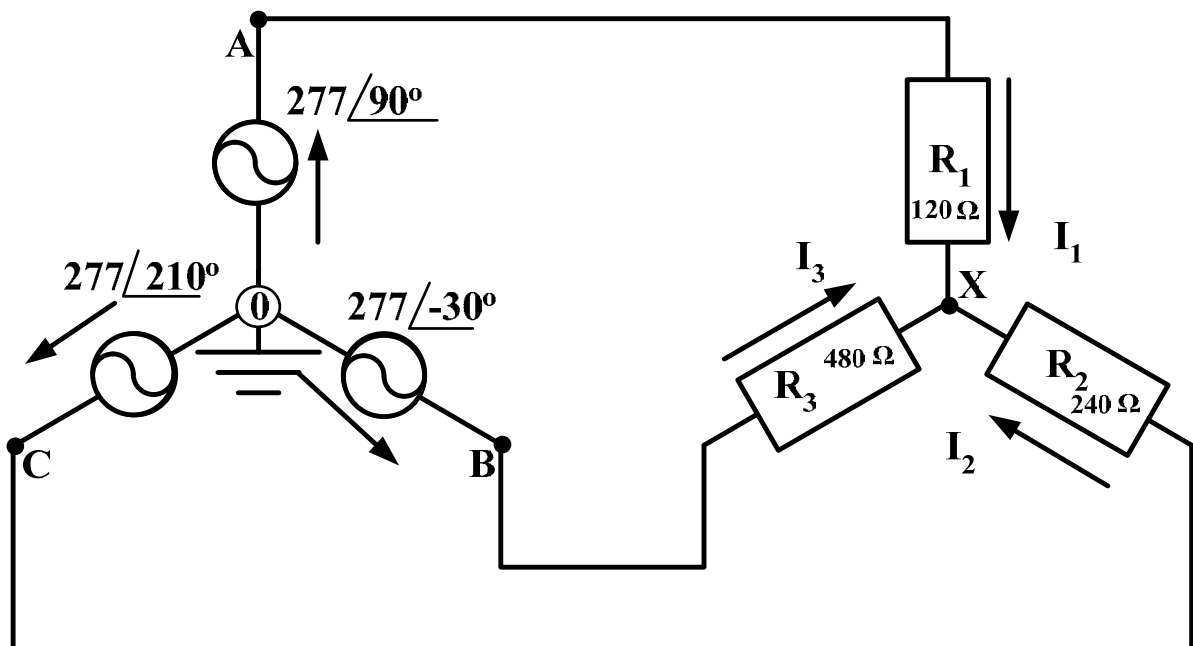
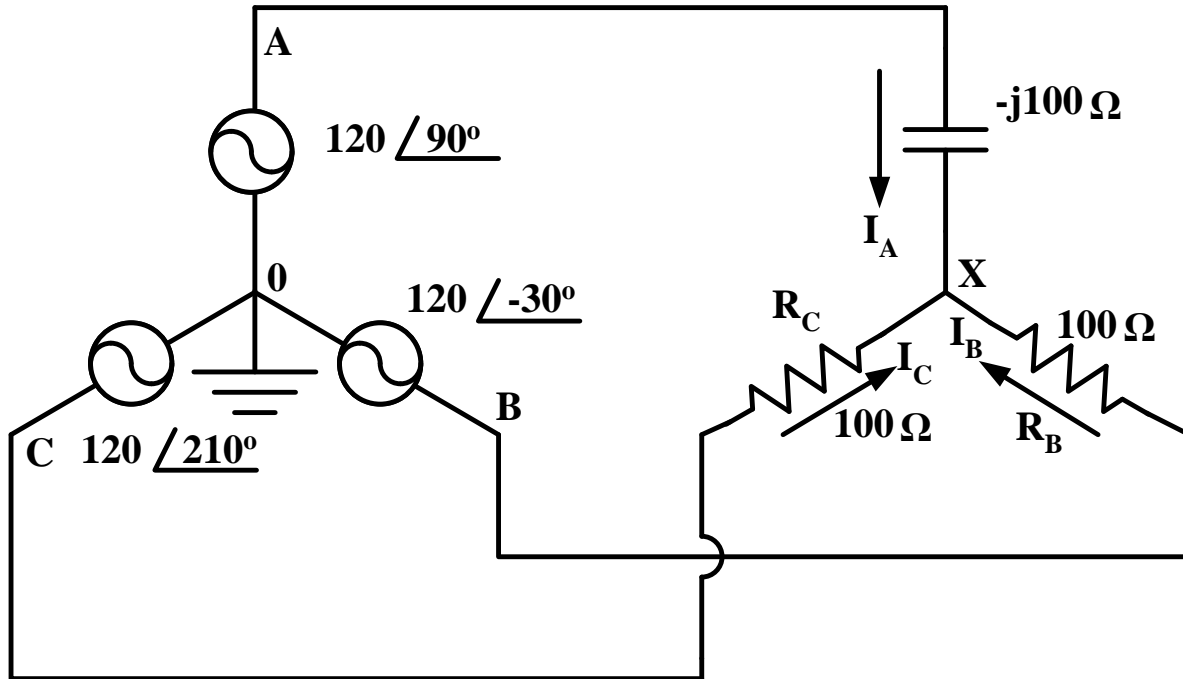


Figure 3.13 The Circuit of Figure 3.12 With the Phase Sequence Reversed for the Load

### 3.6 Phase Sequence Indicator

There is a device called a Phase Sequence Indicator that will indicate the phase sequence of a source. It consists of two resistors (in practice, two light bulbs are used) and a reactive component, usually a capacitor. A schematic drawing of a typical Phase Sequence Indicator is shown in Figure 3.14. Easy to work with values have been chosen for the three components.



**Figure 3.14 Schematic of a Typical Phase Sequence Indicator**

To understand its operation, the concept of capacitive reactance will have to be understood. Instead of having a real resistance, like a resistor, it has an imaginary resistance, or reactance. What really happens is that the current and voltage are not in phase with each other, but for a capacitor, the current leads the voltage. Refer to Figures 2.5 and 2.6 to see this. Another way of looking at this is to see that the reactance of the capacitor is at  $-90^\circ$  away from  $0^\circ$ . Or the reactance of the capacitor is  $-jX_C$ . I picked easy values to analyze for the phase sequence indicator model. You can try this for yourself, but here are the results for the circuit of Figure 3.14.:

$$\begin{aligned}
 V_X &= 75.89 \angle -161.57^\circ \text{ Volts} & I_A &= 1.61 \angle 153.44^\circ \text{ Amps} \\
 I_B &= 1.79 \angle -11.56^\circ \text{ Amps} & I_C &= 0.48 \angle -131.56^\circ \text{ Amps}
 \end{aligned}$$

**Equation 3.15 Solution for the Circuit of Figure 3.14**

Obviously, resistor  $R_B$  has the most current, and if it were a light bulb, it would be brighter. Now if the phase sequence were reversed, as shown in figure 3.15, the solution for the currents would be as follows:

$$\begin{aligned} V_X &= 75.89 \angle -161.57^\circ \text{ Volts} & I_A &= 1.61 \angle 153.44^\circ \text{ Amps} \\ I_B &= 0.48 \angle -131.56^\circ \text{ Amps} & I_C &= 1.79 \angle -11.56^\circ \text{ Amps} \end{aligned}$$

Equation 3.15 Solution for the Circuit of Figure 3.15

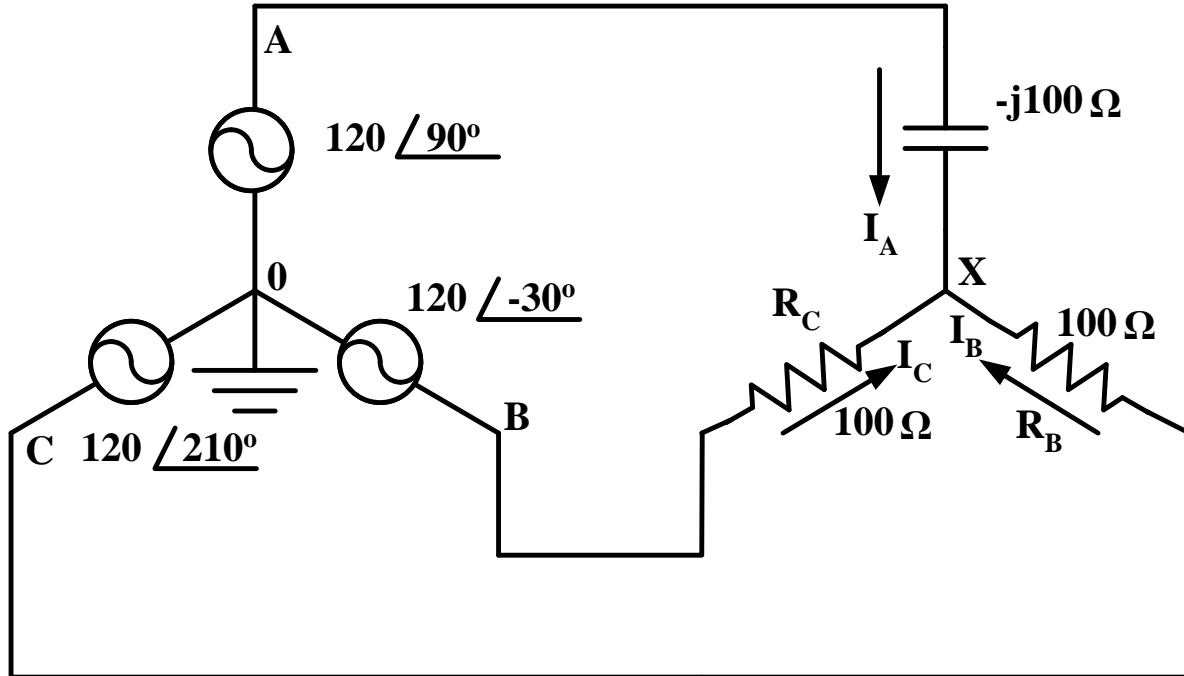


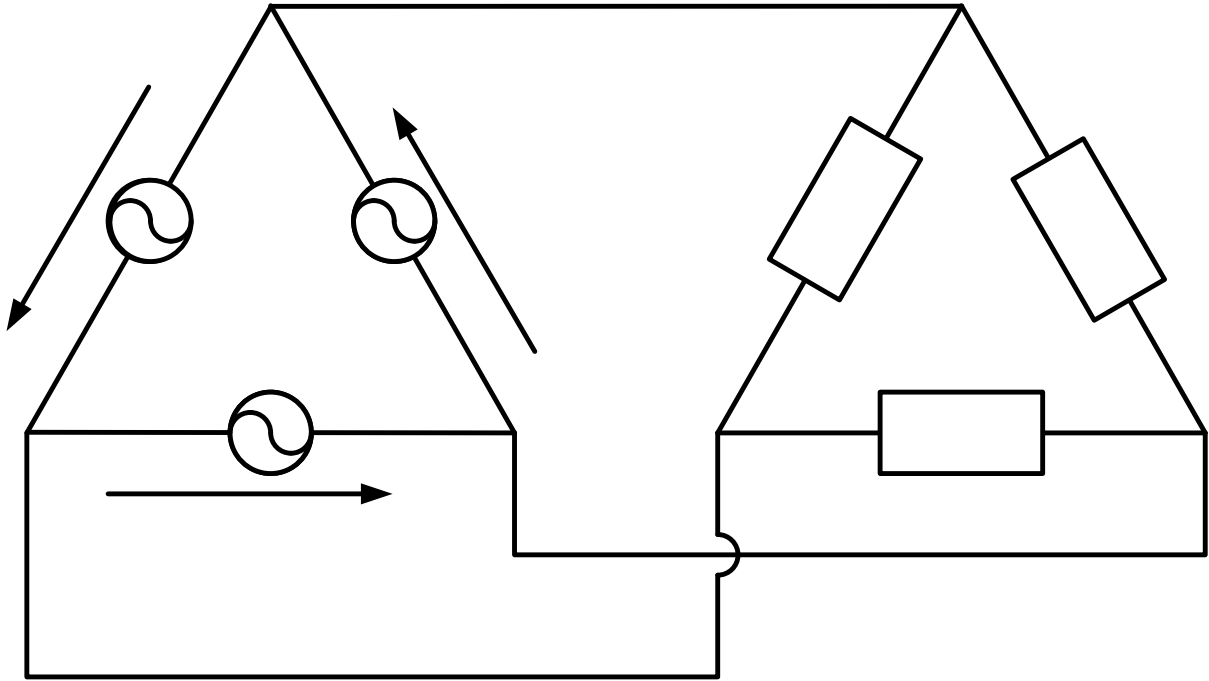
Figure 3.15 Phase Sequence Indicator With Phase Sequence Reversed

Notice that the currents through  $R_A$  and  $R_B$  are the same as before, but are in the opposite resistors. Did you expect the current results for the capacitive reactance, namely that  $I_A$ 's current stays the same for both phase sequences? This makes sense, but it's not readily obvious.

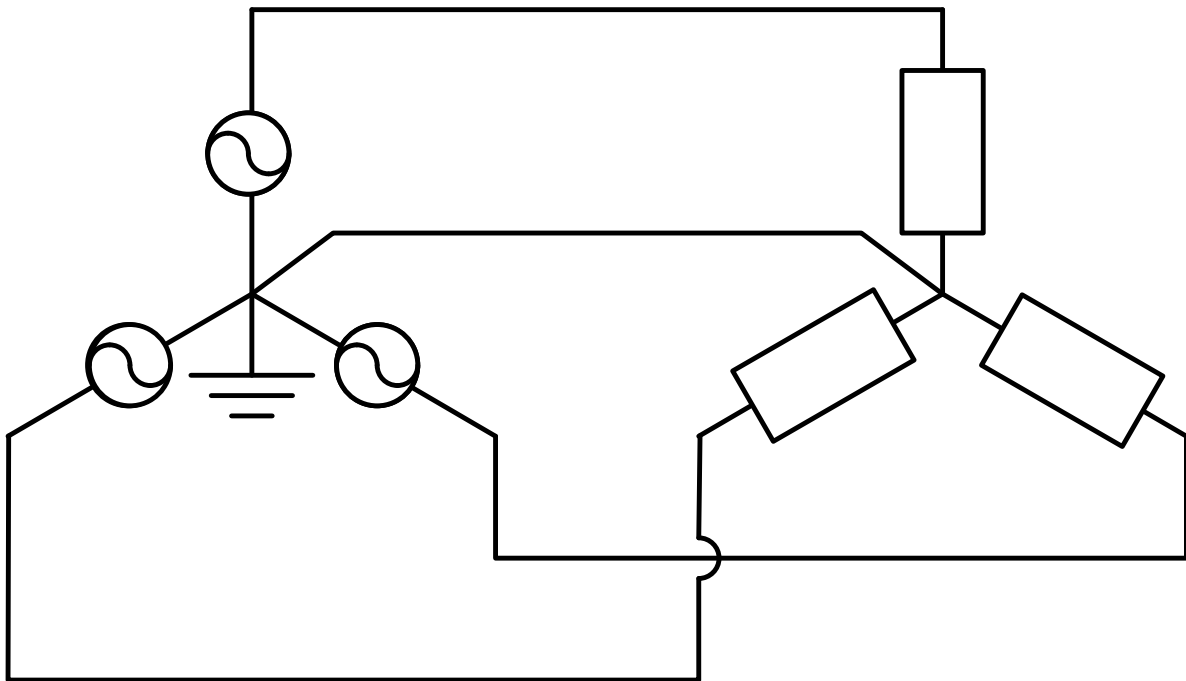
#### 4. Conclusion

I have included with this course 4 templates that will help anyone working with three phase circuits. These cover the most common three phase circuits encountered in the commercial and industrial world. Please feel free to use them to help you in your work.

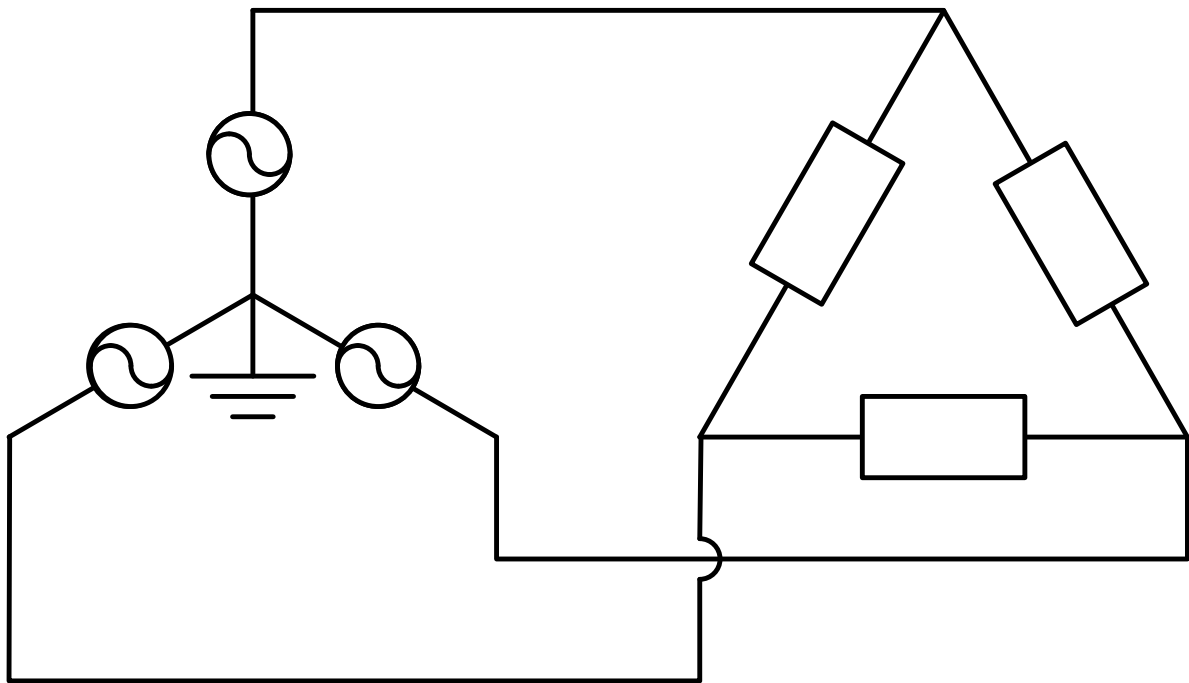
Robert J. Scoff, PE



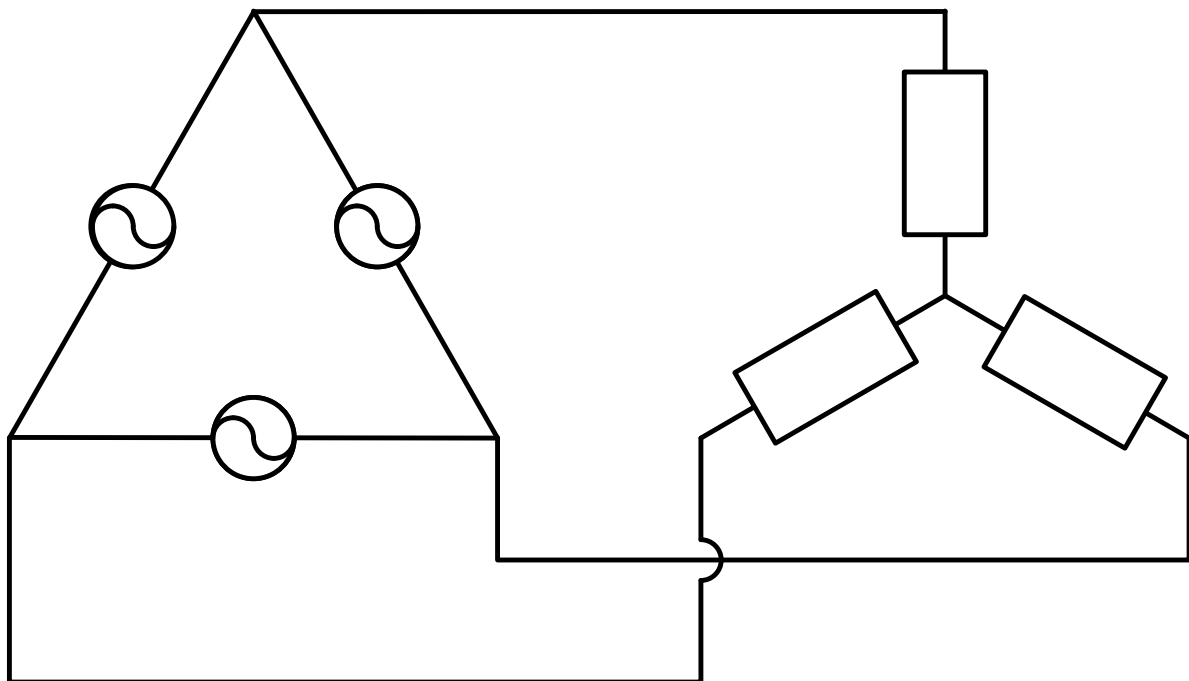
**Delta Delta Three Phase Template**



**Wye Wye Three Phase Template**



**Wye Delta Three Phase Template**



**Delta Wye Three Phase Template**