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PIPE SELECTION AND FRICTION LOSS CALCULATION

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INTRODUCTION

This course is designed to cover the necessary material to design and evaluate piping systems. This course is intended to be one of a series of courses that covers different areas of practical aspects of fluid mechanics. This is the first course of the series and covers the basic areas of water piping, including friction in pipes, diameter selection of pipes and pump power requirements. The next courses will cover more specific piping applications, such as flows in parallel pipes, open and closed piping systems, cooling towers, pipe insulation, and heat exchangers.

These courses are designed to build gradually in concepts. The first course starts with the concept of the difference between compressible and incompressible fluids, and then it covers the concept of fluid friction inside a pipe including the effects of potential and kinetic fluid energy. Once this idea is understood, the calculation of the friction coefficient and the impact of pipe fittings and valves and power through pumps on the overall piping system is introduced. In addition to the course material, the author included a set of calculators in an excel spreadsheet that can greatly simplify the calculation of friction in pipes and reduce the amount of trial and error required to estimate pipe diameters.

Other courses provided by PDHengineer.com cover pumps calculations, so in this course only power requirements for piping systems are considered. If the student is interested in pursuing further studies on pumps, it is suggested that he/she considers any of those fine courses

COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS

In reality all fluids are compressible to a certain degree. However some fluids are very sensitive to variations of pressure and temperatures while others are much less sensitive. For all practical purposes, compressible fluids are those fluids whose density changes 'significantly' over the process. When a fluid experiences a small change in density and that change does not affect in any significant way the final outcome of the process, then the fluid can be considered as an incompressible fluid. For example, designing air ducts for HVAC systems would fall in this category. Even though the air is a compressible fluid, the density changes of the air along the duct are not significant and therefore can be ignored with little error and great simplicity in the calculations. On the other hand, the design of a high pressure compressed air piping system where the density of the air changes, can produce a significant amount of error by considering the air as an incompressible fluid. Good judgment is necessary in most cases.

PRESSURE DROP CALCUATIONS

- **<u>BASIC FLUID DYNAMICS IN PIPES</u>**

As an incompressible fluid flows through a pipe, a friction force along the pipe wall is created against the fluid that will decrease the pressure of the fluid as it moves through the pipe.

The following figure represents a section of fluid inside the pipe. As the fluid flows from left to right, there are a series of forces that act on the element or section of fluid of area A and thickness dx. The conservation of momentum requires that the sum of all the forces equal the change in momentum.

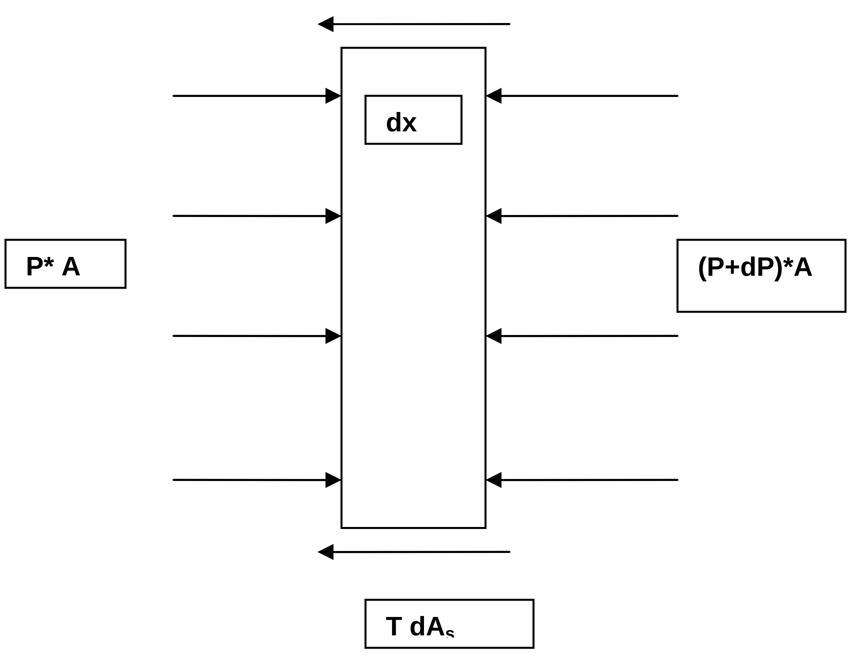


Fig. 1

Since the velocity of an incompressible fluid inside the pipe of a constant diameter is constant, the following equation can be written:

$$P\ A - (P+dP)\ A - T\ dA_s = 0 \qquad \textbf{eqn. 1}$$

Where

P is the initial pressure acting on the fluid element
P+dP is the pressure acting on the other side of the fluid element
A is the cross sectional area of the element
T is the wall shear stress due to friction acting on the fluid
A_s is the peripheral area over which the shear stress acts.

Equation 1 can be written as:

$$A\ dP + T\ dA_s = 0$$

Where

$$A = \pi D^2 / 4$$
$$A_s = \pi D\ dx$$

Where **D** is the element diameter and d**x** is the width of the element.

From here we get the following expression:

$$dP + T\ (4\ dx / D) = 0 \qquad \textbf{eqn 2}$$

In general, if the pipe is not of a circular shape, the value used for the diameter (D) is the hydraulic diameter or Dh and it is defined as:

D_h = (4 * cross sectional area of flow) / (perimeter wetted by fluid)

The following example will illustrate the calculation of the hydraulic diameter in the case of a pipe that is not circular.

Example 1

a) Calculate the hydraulic diameter of a circular pipe of 3" in diameter flowing half full.
b) Calculate the hydraulic diameter of a square pipe of side 3" flowing full.

Solution

a) The cross sectional area of flow of the circular pipe of diameter 3" is $(1/2)\pi D^2/4 = (1/2)9\pi/4$. The perimeter wetted by the fluid is $\pi D/2 = 3\pi/2$.

Using the equation for the hydraulic diameter:

D_h = (4 * cross sectional area of flow) / (perimeter wetted by fluid)

$D_h = (4)(1/2)(9\pi/4)/(3\pi/2) = 3"$ answer

b) The cross sectional area of flow of the square pipe of side 3" is $3^2 = 9$ The perimeter wetted by the fluid is $(4)(3) = 12$

D_h = (4)(9)/12 = 3"   answer

Equation 2 represents the forces acting on the fluid element. In order to represent **T** as a variable in terms of other flow variables and in terms of the friction f , we will use the expression:

$$\mathbf{T = (f / 4) (\rho V^2) / (2 g_c)} \qquad \textbf{eqn 3}$$

Where

V is the mean average velocity of the fluid inside the pipe (ft/sec)
ρ is the fluid density (Lbm/ft^3)
f is the friction coefficient of the fluid in the pipe (dimensionless)
$\mathbf{g_c}$ is the gravity constant

Using these expressions (eqn 2 and eqn 3) we can develop an equation to calculate the pressure drop across a fluid inside a pipe:

$$\mathbf{dP + (f / 4) (\rho V^2) / (2 g_c) (4 dx / D) = 0} \qquad \textbf{eqn 4}$$

Integrating this equation over the length of the pipe it can be rearranged as:

$$\mathbf{P_2 - P_1 = - (\rho V^2) (f L) / (2 g_c D_h)} \qquad \textbf{eqn 5}$$

Where

L is the pipe length (ft)

So far in this development we have used only the effect of the friction of the pipe over the element. If, in addition, there is an elevation difference between point 1 and point 2 or there is a difference in velocity between those same points, this needs to be taken into account.

The final expression developed taken into account the difference in elevation and changes in velocity between any two points is known as the Bernoulli's equation where no work or heat is produced or introduced into the system. It is finally expressed as follows:

$$\mathbf{((P_2-P_1)/\rho) + ((V_2^2-V_1^2)/(2g_c)) + ((g(Z_2-Z_1))/g_c) + ((V^2 f L)/(2g_c D_h)) = 0} \qquad \textbf{eqn 6}$$

Example

If a water flow of 20 cubic feet of water per minute is flowing through a 3 inch pipe, 50 feet long and horizontal, calculate the pressure drop of the water if the friction coefficient of the fluid in the pipe is 0.02 and the water density is 62.4 lbm/ft^3.

Solution
The first thing that we need to do is calculate the velocity of the flow.

$$V = Q / A = 20 / ((1/16) (\pi/4)) = 408 \text{ ft/min or } 6.8 \text{ ft/sec}$$

Once we know the velocity of the fluid in the pipe, we can replace it in eqn 5

$$P_2 - P_1 = -(\rho V^2)(f L) / (2 g_c D_h)$$

$$P2 - P1 = -(62.4 * 6.8^2)(0.02 * 50) / (2 * 32.2 * ¼)$$

$$= -179.2 \text{ Lbs/ft}^2 \qquad \text{answer}$$

Now, assume that there is an incline on the pipe and the discharge point is 5 ft higher than the intake point. To calculate the pressure differential between these two points, we need to include the difference in elevation in the pipe. The formula that we need to use now is the Bernoulli's equation, equation 6. Notice that the velocity in point 1 and 2 are the same since the pipe diameter has not changed.

$$((P_2 - P_1) / \rho) + ((V_2^2 - V_1^2). / (2 g_c)) + ((g Z_2 - g Z_1) / g_c) + ((V^2 f L) / (2 g_c D_h)) = 0$$

since $V_1 = V_2$ the above equation is simplified to:

$$((P_2 - P_1) / \rho) + ((g Z_2 - g Z_1) / g_c) + ((V^2 f L) / (2 g_c D_h)) = 0$$

$$P_2 - P_1 = -\rho ((V^2 f L) / (2 g_c D_h)) - \rho ((g Z_2 - g Z_1) / g_c)$$

$$= -62.4 ((6.8^2 * 0.02 * 50) / (2 * 32.2 * ¼)) - 62.4 ((32.2 * 5) / 32.2)$$

$$= -179.2 - 312 = -491.2 \text{ Lbs/ft}^2 \qquad \text{answer}$$

- **ENERGY REQUIREMENTS TO PUMP A FLUID THROUGH A PIPE**

Again, up to this point we have only considered the pressure drops in a fluid as it flows inside a pipe. The question that we need to address now is: What is the power required to pump the liquid through the pipe? To answer this question we need to look at the conservation of energy equation which states that the energy applied into a system (thermal, mechanical, electrical, etc) minus the energy from the system is equal to the energy stored by the system.

Graphically we can represent this principle in the following figure

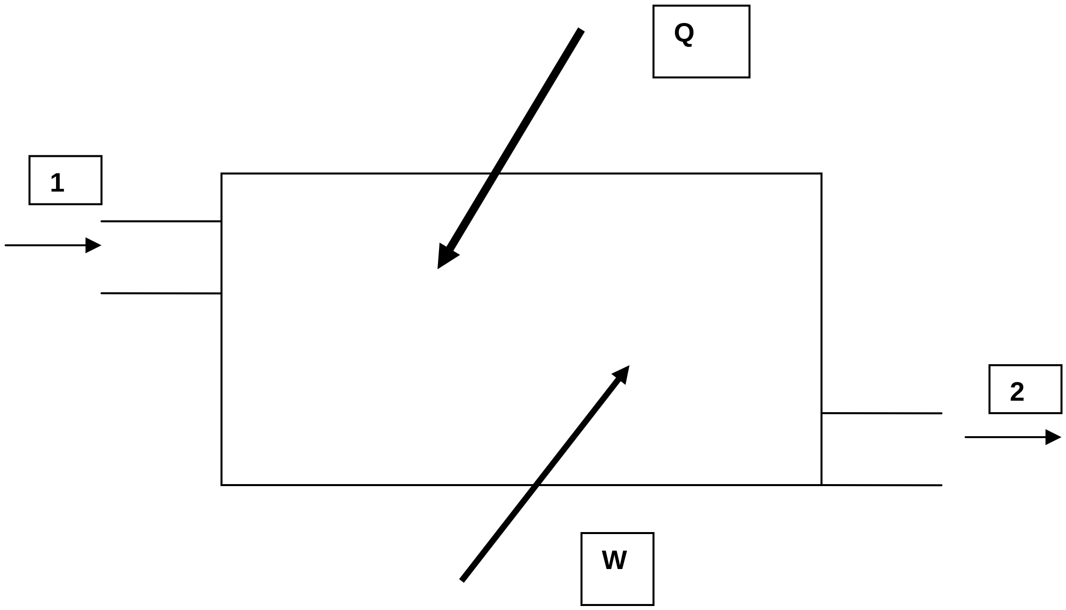


Fig. 2

In this figure we have that a fluid is entering the system through port 1 and leaving through port 2. The velocities at 1 and 2 do not need to be the same. In addition, Heat (represented by Q) is introduced into the system and Work (represented by W) is also introduced into the system. Also, we can see that there is an elevation difference between port 1 and port 2.

The energy balance for this equation can be represented as:

$$d(Q+W)/dt = m\,((h_2+V_2^2/2g_c+Z_2(g/g_c)) - (h_1+V_1^2/2g_c+Z_1(g/g_c))) \quad \textbf{eqn 7}$$

Where

dQ/dt is the heat rate into the system
dW/dt is the work rate into the system
h_2 is the enthalpy of the flow at point 2
h_1 is the enthalpy of the flow at point 1
V_1 is the velocity of the flow at point 1
V_2 is the velocity of the flow at point 2
Z_1 is the elevation of port 1
Z_2 is the elevation of port 2
m is the mass flow rate. Notice that $\rho A_1 V_1 = \rho A_2 V_2$ since the mass flow rate is constant through the system and we are studying incompressible fluids only.

h (enthalpy) = u + P/ ρ

Where

u is the internal energy of the fluid. This energy is dependent on the temperature, therefore in a isothermal process u1 = u2.

When we apply the definition of enthalpy to the eqn 7, and assuming an isothermal process, it becomes:

$$d(Q+W)/dt = \rho AV\left((P_2/\rho + V_2^2/2gc + Z_2(g/gc)) - (P_1/\rho + V_1^2/2gc + Z_1(g/gc))\right) \qquad \textbf{Eqn 8}$$

Example 2

Calculate the power requirement to pump 100 cubic foot per minute of water from a lake to the top of a building at 500 ft of elevation using a 6" pipe. Assume that the length of the pipe is 700 ft, density of the water as 62.0 Lbm/ft^3 and the friction coefficient is 0.02. Neglect the internal energy absorbed by the water. Assume that the pump is located at the surface of the lake.

Solution

This problem will be analyzed in several steps. Much simpler and more direct ways are possible but it is important to consider the individual steps. The solution to this problem is very simple using the software included in the course.
As a first step we will use the energy equation (Eqn 8). If the amount of heat introduced by the pump is negligible, dQ/dt= 0. This equation also applies to inlet and outlet of the pump as points 1 and 2. If the inlet area is the same as the outlet area, the velocities at points 1 and 2 are the same and the equation becomes:

From the lake to the pump we can utilize the Bernoulli's equation:

$$((P_e - P_2)/\rho) + ((V_e^2 - V_2^2)/(2\,g_c)) + (g(Z_e - Z_2)/g_c) + ((V^2\,f\,L)/(2g_c\,D_h)) = 0$$

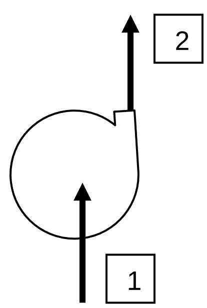


Fig. 3

At the pump we can use the energy conservation equation where the elevations of point 1 and 2 are basically the same and the velocities of the flows at 1 and 2 are also the same if the diameters are the same. In this case all the energy supplied by the pump to the fluid is in the form of pressure rise:

$$d(W)/dt = \rho AV((P_2/\rho)) - (P_1/\rho)) = \rho AV((P_2 - P_1)/\rho)$$

Now we can use the Bernoulli's equation with the pipe friction component (Eqn 6)

$$((P_e – P_2)/\rho)+((V_e^2 – V_2^2)/(2\ g_c))+(g(Z_e – Z_2)/g_c)+((V^2\ f\ L)/(2g_c\ D_h)) = 0$$

Because the pipe is the same diameter (4") at point 2 as it is at point e, we have that $V_2=V_e$.

$$((P_e – P_2)/\rho)+(g(Z_e – Z_2)/g_c)+((V^2\ f\ L)/(2g_c\ D_h)) = 0$$

$$P_2/\ \rho = P_e/\ \rho + g(Z_e – Z_2)/g_c) + ((V^2\ f\ L)/(2g_c\ D_h))$$

Using the Bernoulli's equation from the surface of the lake to the intake of the pump:

$$P_1/\ \rho = P_0/\ \rho - g(Z1 – Z0)/g_c) - (V_1^2 – V_0^2)/(2g_c)$$

Notice that P_0 is atmospheric pressure, that V_0= 0, because point 0 is located on the surface of the lake and that Z_1 is approximately the same as Z_0. Therefore this equation becomes:

$$P_1/\ \rho = P_0/\ \rho - V_1^2\ /(2g_c)$$

Now, substituting these two equations into eqn. 9 to replace the two pressures needed to calculate the power requirements

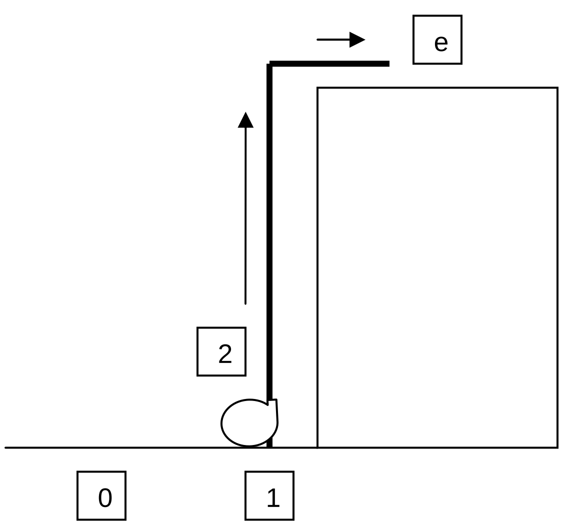


Fig. 4

$$d(W)/dt = \rho AV\ ((P_2 – P_1)/\rho) = \rho AV\ (P_2/\rho) - \rho AV\ (P_1/\rho)$$

$$= \rho AV\ (\ P_e/\ \rho + g(Z_e – Z_2)/g_c) + ((V^2\ f\ L)/(2g_c\ D_h)) - P_0/\ \rho + V^2_1\ /(2g_c))$$

Notice that $V_1 = V_2 = V_e$ and also that $P_0 = P_e$ (atmospheric pressure). Also notice that $Z_0=Z_1=Z_2$ in relationship to the total elevation of the building. Now we can calculate the power of the pump as a function of the location of the lake and the building.

Calculating the velocity of the water:

$$V = 100 / ((\pi/4)(9/144)) = 509.3 \text{ fpm} = 8.48 \text{ fps}$$

The mass flow is calculated as

$$M = (100/60)(62.0) = 103.3 \text{ Lbm/sec}$$

Notice that $M = \rho AV$

Now substitute these values into equations 8 and 6

$$d(W)/dt = (103.3)*(0 + 500 + ((0.02)(700)(8.48^2) / ((1/2)(2)(32.2))) + ((8.48^2) / ((2)(32.2)))$$

$$=54{,}835 \text{ ft-Lbf / sec}$$

If 550 ft-Lbf / sec = 1 Hp:

Power required by the pump = 99.7 Hp answer

Discussion of the problem

Notice that in reality the exact location of the pump would not affect the result of the problem as the pipe length is the same regardless of the pump location since the overall height that the pump has to pump the water is given only by the difference between Z_e and Z_0. The method of solution of the problem was chosen to highlight the effect of the pump alone, and the effect of the pipe friction.

- **CALCULATION OF FRICTION COEFFICIENTS IN PIPES**

Up to this point we have discussed the flow equations in pipes (momentum and energy). We have also included the equations required to calculate the power requirements. We have also assumed values for the friction coefficient when we did the calculations. This section will cover the calculation of the friction coefficient for pipes.

The coefficient of friction in a fluid, depends on shear stress of the wall, which is defined as:

$$T = \mu \, (dV/dr)$$

Using this expression and the equations 2 and 3, it can be proved that the velocity profile of a laminar flow is parabolic. Many text books in Fluid Mechanics have this development and it is not the intent of this course to prove this type of behavior. As the fluid inside the pipe increases its turbulence, the velocity profile starts to change and it becomes more irregular. As the velocity profile changes, so does the coefficient of friction. Therefore, it is very important to know what type of flow is in the pipes, laminar, transitional or fully turbulent.

To calculate the pressure drop of a fluid inside a pipe alone, we use the eqn. 6 for constant velocity and no pipe elevation:

$$((P_2 - P_1) / \rho)) + ((V^2 f L) / (2 g_c D_h)) = 0 \qquad \text{Eqn. 9}$$

One measurement used to determine the type of flow is the **Reynolds Number**. The Reynolds number is a dimensionless number that is directly proportional to the turbulence of a flow. It is defined as:

$$Re = \frac{V D_h \rho}{\mu g_c} \qquad \text{Eqn. 10}$$

Where:

V is the average fluid velocity in the pipe (ft/sec)
D_h is the hydraulic diameter of the pipe (ft)
ρ is the fluid density Lbm/ft^3
μ is the dynamic viscosity of the fluid (Lbf-sec/ft^2)

Another expression used for the Reynolds Number is using the kinematic viscosity instead of the dynamic viscosity:

$$Re = \frac{V D_h}{v} \qquad \text{Eqn. 11}$$

Where

v is the kinematic viscosity of the fluid (ft^2/sec)

Another variable that is critical in the calculation of the friction coefficient is the pipe roughness. The pipe roughness is a characteristic of the type of pipe used. For example a glass galvanized iron pipe has a pipe roughness of 0.0005 ft while a cast iron pipe has roughness of 0.00085 ft. Other types of materials or surface finishes have different pipe roughness.

The absolute roughness of a pipe would not have a significant meaning on the overall friction if we don't specify the inside diameter of the pipe. So, the variable that we need to take into account is the **Relative Roughness** of the pipe, which is defined as:

$$\text{Relative Roughness} = \frac{\varepsilon}{D_h} \qquad \text{Eqn. 12}$$

Where:

ε is the pipe roughness characteristic of the material or pipe finish (ft)
D_h is the hydraulic diameter of the pipe (ft)

The number is also dimensionless and therefore can be used as an absolute number as long as the units are consistent.

A table of pipe roughness for several types of pipes is included in the Appendix of this course.

In order to calculate the friction coefficient of a flow inside a pipe we need to use the Moody diagram. This diagram is a graph that correlates the Reynolds number with the Relative roughness and provides a friction coefficient. A copy of the Moody diagram is attached to the Appendix of this course and can be found in many sources of Fluid Mechanics textbooks.

Notice that the Moody diagram shows the laminar region (Reynolds numbers less than 2300) and the friction coefficient as a linear relationship between Reynolds number and friction coefficient. As the flow starts getting more turbulent (Reynolds numbers greater than 2300) the friction coefficient starts becoming more and more dependent on the Relative roughness of the pipe. Eventually, when the Reynolds numbers are higher than $2(10^7)$ the friction coefficient is basically independent of the Reynolds number and dependent only on the Relative roughness of the pipes. This last transition occurs at lower Reynolds numbers for rougher pipes.

One area that the student should be aware of is the fact that fluids do not develop the velocity profile immediately after they enter a pipe. The friction coefficient also varies as it enters the pipe. It takes sometimes over an equivalent of 10 diameters of pipe distance to fully develop into a turbulent flow and somewhat greater than that is required to fully develop a laminar flow. For these reasons, unless the pipes are extremely short, the entrance effect into a pipe can be neglected with a very small error but greater simplicity.

- **Comments on the Moody Diagram.**

A careful study of the Moody diagram by the student is recommended. Notice that for very small Reynolds numbers, the flow is laminar and the friction coefficient is very high and it is also independent of the type of pipe that is used. In this region, the friction coefficient is calculated as Re/64 and this equation is valid for Reynolds numbers that are less than 2300.

As the Reynolds number increases, the flow enters a transition zone where friction coefficient is dependent on the pipe roughness and the Reynolds number. There are several equations in the literature that can represent the values of the friction coefficient in this region. The complexity of this region requires that some of those equations be solved using trial and error techniques.

Finally, when the flow is fully turbulent, we can see that the Relative Roughness are basically horizontal and therefore, the friction coefficient does not depend on the value of the Reynolds number, but basically only on the value of the Relative Roughness of the pipe. This is more pronounced as the Relative Roughness increases in value.

Solving problems using this diagram will require trial and error techniques some times. This is particularly true when we need to calculate or estimate a pipe diameter. Some example problems included in this course illustrate this point. In addition, this course includes a simple calculator that will greatly simplify this work and makes the trial and error solutions very simple and fast.

The following example will illustrate the calculations of the pressure drop using the Moody diagram:

Example

Compare the pressure drop of two circular pipes of 3" in diameter and 1000 ft of horizontal length that carry 100 gallons per minute of water at 80 F. One pipe is made of cast iron and the other is made of galvanized steel.

Solution

First we need to determine the velocity of the water in the pipe. Notice that 100 GPM is equivalent to 13.4 ft^3/min

$$V = 13.4 / ((\pi/4)(1/16)) = 273 \text{ ft/min} = 4.55 \text{ ft/sec}$$

From the tables in the appendix, the dynamic viscosity of the water at 80 F is 1.81 (10^{-5}) lbf-sec/ft^2. The density of the water at that temperature is 62.2 lbm/ft^3

Using these values into the Reynolds number:

$$Re = (62.2/32.2)(4.55)(1/4)/(1.81(10^{-5}) = 1.22\ (10^5)$$

Note that in order to keep the Reynolds number dimensionless, it was necessary to divide the Reynolds number by 32.2 Lbm-ft/Lbf-sec^2

Now, for the cast iron pipe the pipe roughness is 0.00085 as it can be read from the Appendix.

$$\varepsilon / D = (0.00085)(4) = 0.0034$$

From the Moody diagram using the relative roughness and the Reynolds number, we can read the value of f = 0.028.

The pressure drop is calculated using eqn. 6

$$((P_2 - P_1) / \rho) + ((V^2 f L) / (2 g_c D_h)) = 0 \quad \text{or,}$$

$$P_2 - P_1 = -\rho ((V^2 f L) / (2 g_c D_h)) = -62.2 * ((4.5^2)(0.028)(1000) / ((64.4)(1/4))$$

$$= -2{,}240 \text{ Lbf/ft}^2 = 15.55 \text{ psi} \qquad \text{answer}$$

For the galvanized pipe, the pipe roughness is 0.0005 ft, so the relative roughness is calculated as

$$\varepsilon / D = (0.0005)(4) = 0.0020$$

From the Moody diagram we can read the value of f = 0.025.

In a similar manner we calculate the pressure drop as:

$$P_2 - P_1 = -\rho ((V^2 f L) / (2 g_c D_h)) = -62.2 * ((4.5^2)(0.025)(1000) / ((64.4)(1/4))$$

$$= -2000 \text{ Lbf/ft}^2 = 13.9 \text{ psi} \qquad \text{answer}$$

This example illustrates the difference of piping materials in the pressure drop of the fluid.

It is important to keep in mind that the values of ε in the Appendix Table T-1 are for new pipes only. As time goes on, the pipes tend to scale up and corrode. As the corrosion builds up, the values of the relative roughness of the pipes also changes. The change in relative roughness of a pipe can be expressed as indicated in the following expression:

$$\varepsilon = \varepsilon_0 + k\,t \qquad \textbf{Eqn. 13}$$

Where

ε_0 is the initial pipe roughness (ft)
k is the proportional constant for the specific pipe (ft/year)
t is the time that the pipe has been used. (years)

- **PRESSURE DROP DUE TO FITTINGS AND VALVES IN A PIPE**

Again, up to this point we have covered the concept of friction and the effects of friction on a fluid that is flowing inside a pipe. We have also covered the effects of pipe elevation and fluid velocity. Now we will expand the study to cover the impact that pipe fittings (elbows, "T", unions, etc) and valves will have on the final piping design.

When a fluid travels through a pipe fitting or a valve, it creates a series of eddy currents that tend to increase its pressure drop. These eddy currents take away some of the energy that the fluid has and causes the pressure drop. The amount of energy drop or pressure drop across a fitting is proportional to the square velocity of the fluid. The expression can be represented as:

$$\Delta P = - K\,(\rho)(V^2) / ((2)(g_c)) \qquad \textbf{Eqn. 14}$$

Where:

K is constant dependent on the type of fitting or valve (dimensionless)
V is the fluid velocity across the fitting or valve (ft/sec)
ρ is the fluid density (Lbm/ft^3)

In table T-2 in the Appendix there is a list of several values of K for different fittings and valves

This expression needs to be added to the Bernoulli's equation (eqn. 6) to include the effects of the pipe fittings on the equation:

$$((P_2 - P_1)/\rho) + ((V_2^2 - V_1^2)/2g_c) + ((g(Z_2 - Z_1)/g_c) + ((V^2 f\,L)/(2g_c D_h)) + (K(\rho)(V^2)/2g_c) = 0 \qquad \textbf{Eqn 15}$$

In areas where there are two pipe diameters, such as a pipe expansion or contraction, the K factor is proportional to the ratio of areas. In this case, the velocity is calculated as a function of the ratio of the areas. This is particularly true when you deal with incompressible fluids, which is the scope of his course.

The following example will illustrate the use of the Bernoulli's equation when there is a fluid that flows through a pipe that has several fittings and finally discharges to the atmosphere.

Example

Calculate the water flow rate of a pipe that is connected to the bottom of a cooling tower. The pipe is 3" in diameter, 50 ft long and is made out of galvanized steel. The water that flows from the cooling tower discharges freely to a tank. See fig. 5

Solution

In this example we have a square edge entrance, two std 90 degree elbows, a globe valve open and a free discharge to the tank. Notice that the pressure at the discharge of the pipe is the same as the pressure on the water level in the cooling tower (atmospheric pressure in both cases).

It is also important to notice that the variable that we need to find in order to solve the problem, (V), is also needed to determine the friction coefficient of the pipe. A detailed solution to this problem is explained below.

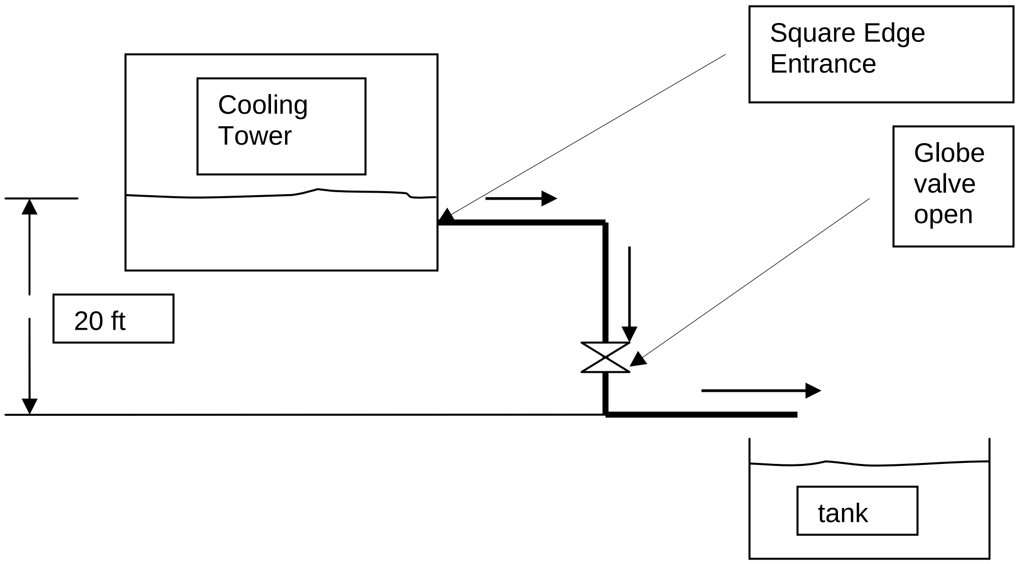


Fig. 5

Using the Bernoulli's equation from the water level in the cooling tower and the discharge point, and adding the fittings term, we can write:

$$((P_e - P_0)/\rho)+((V_e^2 - V_0^2)/(2\ g_c))+(g(Z_e - Z_0)/g_c)+((V^2\ f\ L)/(2g_c\ D_h))+\Sigma\ K(V^2/(2g_c)) = 0$$

Since we are dealing with water (incompressible fluid), $V = V_e$. Also V_0 is zero or extremely small compared to V since the tank surface area is much greater than the cross sectional area of the pipe.

$Z_e - Z_0 = 20$ ft
$V_0 = 0$
$P_e = P_0$

Next thing we can do is identify the K values for the elbows and the globe valve when it is open. These values can be taken from the Appendix table T-2. $K_{elbows} = 0.75$, $K_{open\ globe\ valve} = 6.4$, $K_{square\ edge\ entrance} = 0.5$

Using these values, the Bernoulli's equation becomes:

$$(V^2)/(2\ g_c))+(g(Z_e - Z_0)/g_c)+((V^2\ f\ L)/(2g_c\ D_h))+\ (V^2/(2g_c)\Sigma\ K) = 0$$

$$20 = ((f\ L/D_h)+1+(0.5+6.4+1.5))\ (V^2)/(2\ g_c))$$

Replacing the other values:

$$20 = ((f\ (50)/(1/4))+9.4)\ (V^2)/(2\ g_c))$$

In order to calculate the friction coefficient factor, we need to first calculate the Reynolds number and then use the pipe roughness to look in the Moody diagram and estimate the friction coefficient factor.

The problem that we have here is that the Reynolds number is a function of the fluid velocity and the fluid velocity is basically what the problem is asking for (water flow rate = A V).

In order to solve the problem we need to assume a value for the friction factor (f) and see if that number was close enough for our calculations, if not, we need to assume another value and evaluate again. A trial and error process is very tedious and can take a significant amount of time to solve. To help with this, the student is provided with an Excel calculator that can solve trial and error problems very quickly and with a good accuracy (within 1-3%) which in most practical cases is more than sufficient.

Manually, assume f = 0.02 as a first trial. At a relative roughness of the pipe ε = 0.0005/(1/4) = 0.002, enter the Moody diagram and read the Reynolds number. At these values of f = 0.020 and ε = 0.002, we can see that the graphs don't cross and therefore we need to choose a higher value of f.

As a second trial, assume now f = 0.028. Substituting these values into the equations:

$$20 = (((0.028)\ (50)/(1/4))+9.4)\ (V^2)/(2\ g_c)) = 15\ (V^2)/(2\ g_c)$$

$$V = ((20)(64.4)/15)^{1/2} = 9.26 \text{ ft/sec}$$

$$Re = (9.26)(1/4)/(1.04*10^{-5}) = 2.22*10^5$$

Even though we are closer to the answer as it can be seen in the Moody diagram, we still need to get a better assumption for f. Assume now, f = 0.024.

$$20 = (((0.024)\ (50)/(1/4))+9.4)\ (V^2)/(2\ g_c)) = 15\ (V^2)/(2\ g_c)$$

$$V = 9.5 \text{ ft/sec}$$

$$Re = (9.5)(1/4)/(1.04*10^{-5}) = 2.28*10^5$$

This number matches the Moody diagram quite well and therefore it can be considered to be a good estimate and proper answer for the value of f. The water flow rate is calculated as:

Water flow rate = A V. = (π/4)(1/16)(9.5) = 0.466 ft^3/sec = 27.8 ft^3/min   answer

When we need to find the diameter of the pipe, a similar problem occurs. In a way it is slightly more complicated because we cannot calculate the Reynolds number or the relative roughness of the pipe. A similar solution of trial and error is required in this case. Once again, a simple Excel program can be used to do several trial and error situations in a very small amount of time. In many cases the solution can be as accurate or better than using the Moody diagram.

The next example will also include the impact of a pump to move a certain amount of water through a pipe that has a few fittings. In this case we need to calculate the pipe diameter. This example will cover the most general case of a fluid problem covered in this course. Following courses by this author will cover additional topics such as parallel pipes, pipe insulation, cooling towers and closed and open water systems which are widely used in cooling processes.

Example

A 50 hp pump is used to pump 900 GPM of cooling water at 50 F from a tank A to a process tank B located 400 ft away in a straight line. The two tanks are at the same level. Estimate the minimum size cast iron pipe diameter needed.

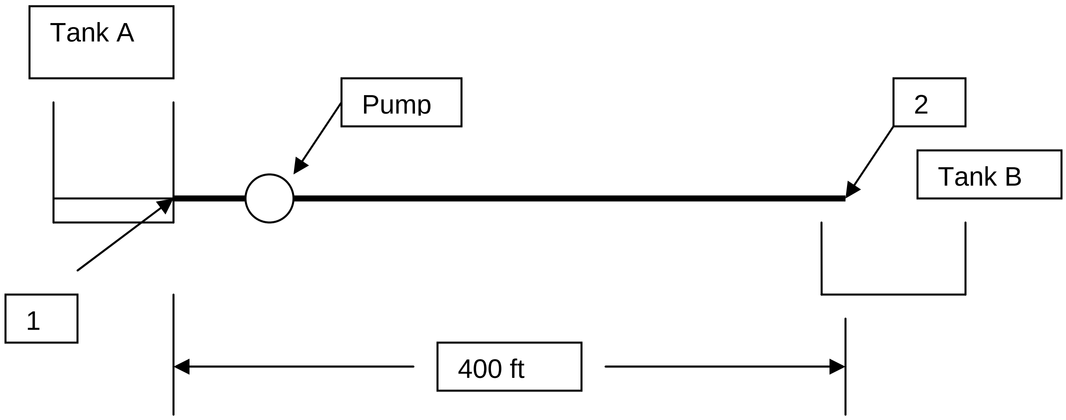


Fig. 6

Solution

In this example we can use the Bernoulli's equation with an external source of energy to pump the water through the pipe.

$$d(Q+W)/dt = -\rho AV\ ((P_2-P_1)/\rho)+((V_2^2-V_1^2)/(2g_c))+((g(Z_2-Z_1)/g_c)+((V^2 f\ L)/(2g_c D_h))$$

Considering that the amount of heat transferred to the fluid through the pipe is negligible, dQ/dt=0. The problem is also telling us that $P_1=P_2=P_{atmospheric}$, $Z_1=Z_2$ and $V_1=V_2$, so when we use these values in the above equation:

$$d(W)/dt = -\rho AV\ ((V^2 f\ L)/(2g_c D_h))$$

$$d(W)/dt\ /\ \rho AV = -((V^2 f\ L)/(2g_c D_h))$$

Note that 900 GPM = 2.01 ft^3/sec

$$(50\ hp)(550\ ft\text{-}Lbf/sec\text{-}hp)\ /\ (2.01\ ft^3/sec)(62.4\ Lbm/ft^3) = 219.25\ ft\text{-}Lbf/Lbm$$

$$219.25\ ft\text{-}Lbf/Lbm = -((V^2 f\ L)/(2g_c D_h)) \qquad \text{Eqn.}$$

Now, in this case we have three variables to solve: V, f, D_h. Therefore, what we need to do is to assume a pipe diameter and check if the assumption is within the normal limits of allowable error. If the error is too great then we will assume another pipe diameter and check again.

In this case we will start the trial and error process assuming a value for D_h = 5 inches.

$Re = 4.35*10^5$ $\qquad \varepsilon = 0.0020$ $\qquad V = 14.74$ ft/sec

$f = 0.024$

Substituting these values in the equation above we can check:

– $((V^2 f\ L)/(2g_c D_h)) = ((14.74^2)(0.024)(400)/((64.4)(5/12)) = 77.73$ Lbf/Lbm

This number is significantly smaller than the required number of 219.25 Lbf/Lbm, therefore we need to assume a different pipe diameter. Now assume a pipe diameter of 4 inches:

$Re = 5.3*10^5$ $\varepsilon = 0.0025$ $V = 21.93$ ft/sec

$f = 0.025$

$((V^2 f\ L)/(2g_c D_h)) = ((21.93^2)(0.025)(400)/((64.4)(4/12)) = 224.03$ Lbf/Lbm

This answer is close enough to the solution of the problem.

The calculator included in this course will greatly simplify this process as the student only needs to enter different values of D_h until the answer is obtained.

A sample figures of the calculators included in this course shown in the section FLOW CALCULATORS

- **PRESSURE DROP DUE TO CHANGES IN PIPE DIAMETER**

When the diameter of the pipe changes in a system, we need to take into account this change and consider this as a fitting in the pipe. The values of the K constant for this type of fitting will vary depending on the ratio of the two areas across the sudden expansion or contraction.

The value of K for a sudden expansion is calculated as:

$$K = (1 - A_1 / A_2)$$

For example, if we are expanding to a tank where $A_2 >> A_1$, K = 1.0. This value can slightly be modified as the curvature of the expansion changes.

In the case of a sudden pipe contraction, the values of K can calculated more experimentally and are included in the table below:

A_2 / A_1	K
0.1	0.37
0.2	0.35
0.3	0.32
0.4	0.27
0.5	0.22
0.6	0.17
0.7	0.1
0.8	0.06
0.9	0.02
1.0	0

Table 1

The pressure drop across a contraction is calculated from the following equation:

$$\mathbf{\Delta P = - K \rho V_2^2 / 2g_c} \qquad \textbf{Eqn. 16}$$

In this case, if $A_1 = A_2$, we see that K = 0. On the other hand, if there is a contraction from a pipe to a tank where $A_1 >> A_2$, K is approximately 0.5. This value is very dependent on the round edge and how few eddy currents are generated in the process.

Example

Calculate the pressure drop across a horizontal galvanized steel pipe that starts at 6" in diameter and after 100 ft it changes to 3" in diameter for another 200 ft. Assume that water at 50 F flows through the pipe at 250 GPM.

Solution

First we calculate the ratio of the areas $A_2 / A_1 = 9/36 = 0.25$. From the table ___, we can read that K = 0.33

Using the Moody diagram (or the software included in this course), we can find that the friction coefficient for the 6" pipe is:

The velocity of the fluid is 2.84 ft/sec, the Reynolds number is $1.01*10^{-5}$ and the friction coefficient is 0.022.

Using the Bernoulli's equation, we can calculate the pressure drop across the 6" pipe, using $V_1=V_2$ and $Z_1=Z_2$:

$$((P_1–P_0)/\rho)+((V^2 f L)/(2g_c D_h)) = 0$$

$$P_1–P_0 = \rho ((V^2 f L)/(2g_c D_h)) = (62.41)(2.84^2)(0.022)(100)/(64.4*1/2) = 34.39 \text{ Lbf/ft}^2$$

Using the same method for the 3" pipe but adding the contraction fitting into the friction equation:

In this case the velocity of the fluid is V =11.38 ft/sec and the Reynolds number is $2.01* 10^{-5}$. From here we can estimate the friction coefficient as 0.0243.

$$((P_1–P_0)/\rho)+((V^2 f L)/(2g_c D_h)) + (K V_2^2 / 2g_c) = 0$$

$$P_1–P_0 = \rho (((V^2 f L)/(2g_c D_h)) + (K V_2^2 / 2g_c)) =$$

$$(62.41)((11.38^2)(0.0243)(200)/(64.4*1/4) + ((0.33)(11.38^2)/(64.4)) = 2481 \text{ Lbf/ft}^2$$

Therefore the final pressure drop across the total pipe is the sum of both pressure drops.

Total pressure drop = 2515 Lbf/ft^2 answer

- **RECOMMENDED FLUID VELOCITIES IN PIPES**

Designing for the proper fluid velocity in a pipe is critical since it affects the overall project cost. In most cases the engineer needs to be aware of the cost trade off between the initial and operational cost. A large pipe diameter will increase the initial cost but will reduce the operating cost. Both costs need to be considered in the project and the minimum needs to be selected.

Maximum velocities are established by:

a) Noise generated by the water flowing through the pipe
b) Erosion cased by the water and the sand or other particles flowing through the pipe

Based on the above criteria, typical water velocities inside a pipe should kept to approximately between 3 and 10 feet per second. These velocities can be different depending on the type of pipe used or in the specific application.

- **FLOW CALCULATOR**

The following figures are samples of excel calculators that can be used to solve the problems. These calculators are included in the purchase of this course.

Moody Diagram -- Friction Coefficient

<u>**INPUT**</u>

Gpm	200
Inside Diam	4
Pipe Roughness	0.00085
Density	62.4
Dynamic Viscosity	0.0000181

<u>**OUTPUT**</u>

Diameter in ft	0.333
Velocity	5.120
Reynolds Number	1.83E+05
Flow Rate	0.4468
Area	0.087
Relative Roughness	0.0026

Friction Coeff.	0.026

POINT 1

Enter data

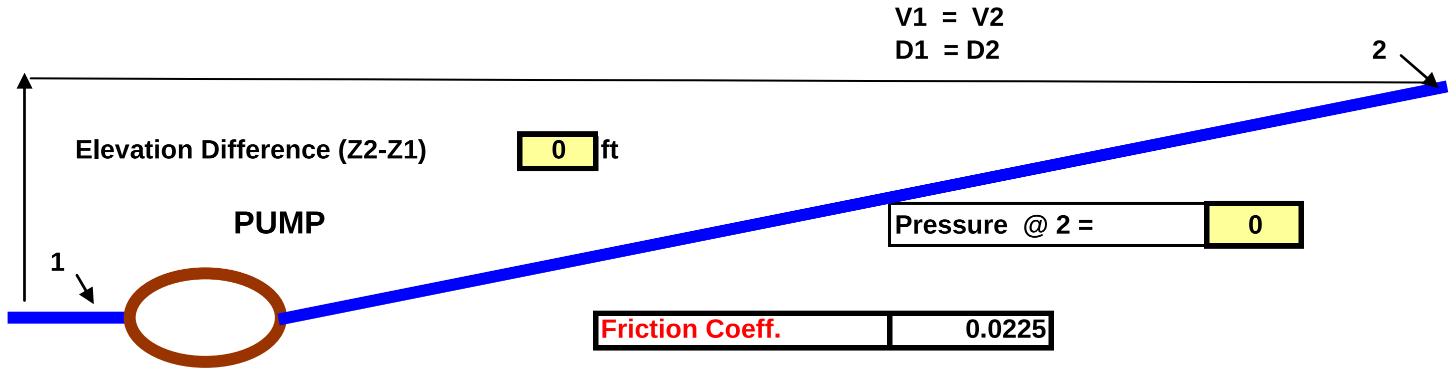
Read results

Enter dat (Units are in comment boxes)

(Units are in comment boxes)

Single Pump Pipe Calculator

Total Head 1 + Head of Pump - Friction Head - Fittings Head = Total Head 2



Friction Coeff.	0.0225

Pressure @ 1	0	psi

Total head 1	0.00	ft
Total head 2	0.00	ft
Friction Loss for Fitting	0.00	ft
Friction Loss for Lengt	246.99	ft
Friction Loss Across C	0.00	ft
Head of Pump =	247.0	ft
	106.9	psi
POWER	20663.4	Ft-Lbf/sec
HP	37.6	HP

Gpm	600
Inside Diam	4
Pipe length=	1000
Fluid	water
Pipe Roughne	0.0005
Density	62.41
Dynamic Visc	2.74E-05

Velocity	15.36
Reynolds Number	3.62E+05
Flow Rate	1.3405
Pipe Area	0.0873
Relative Roughness	0.0015
Diameter in ft	0.3333
Mass Flow Rate	83.66

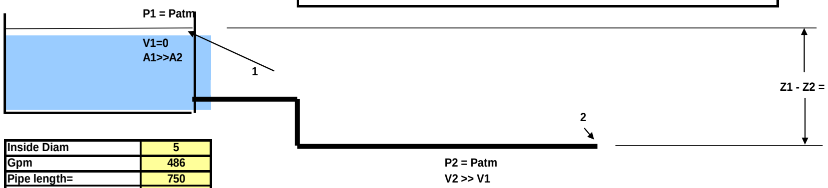
Fittings and Valves @ Diameter			
		Numbers	
Std 45 elbow	0.35		0.00
Std 90 elbow	0.75	0	0.00
Long Radius	0.45		0.00
Coupling	0.04		0.00
Union	0.04		0.00
Gate valve op	0.2	0	0.00
Gate valve 3/4	0.9		0.00
Gate valve 1/2	4.5		0.00
Gate valve 1/4	24	0	0.00
Globe valve o	6.4	0	0.00
Gobe valve 1/	9.5		0.00
"T" line flow	0.4	0	0.00
"T" branch flo	1.5	0	0.00
Rapid Contac	0.33	0	0.00
Entrance	0.5		0.00
	Total (ft)	=	0.00

GRAVITY PIPE FRICTION / FLOW CALCULATOR

Total Head 1 + Balance - Friction Head - Fittings Head = Total Head 2

When Balance=0, you find the proper equilibrium in the equation and it is the answer to the problem
This is particularly helpful when you need to use trial and error techniques

Pressure at point 1	14.7	psi



Z1 - Z2 = Elevation Difference	-45

Inside Diam	5
Gpm	486
Pipe length=	750
Pipe Roughness	0.0004
Density	62.2
Dynamic Viscosity	1.81E-05

Total head 1	34.03	ft

Friction Coeff.	0.020

Flow Velocity	7.96
Flow Rate	1.0858
Reynolds Number	3.54E+05
Pipe Relative Roughness	0.0010
Mass Flow Rate	67.54

Total head 2	-9.98	ft
Friction Loss for Fittings	3.93	ft
Friction Loss for Pipe Length	36.20	ft
Friction Loss Across Others	0.00	ft

Balance	0.052	ft

Fittings and Valves @ Diameter

		Numbers	
Std 45 elbow	0.35		0.000
Std 90 elbow	0.75	3	0.957
Long Radius 90 elbow	0.45		0.000
Coupling	0.04		0.000
Union	0.04		0.000
Gate valve open	0.2	1	0.085
Gate valve 3/4 open	0.9		0.000
Gate valve 1/2 open	4.5		0.000
Gate valve 1/4 open	24		0.000
Globe valve open	6.4	1	2.722
Gobe valve 1/2 open	9.5		0.000
"T" line flow	0.4	1	0.170
"T" branch flow	1.5	0	0.000
			0.000
		Total (ft)	3.934

APPENDIX

Moody Diagram

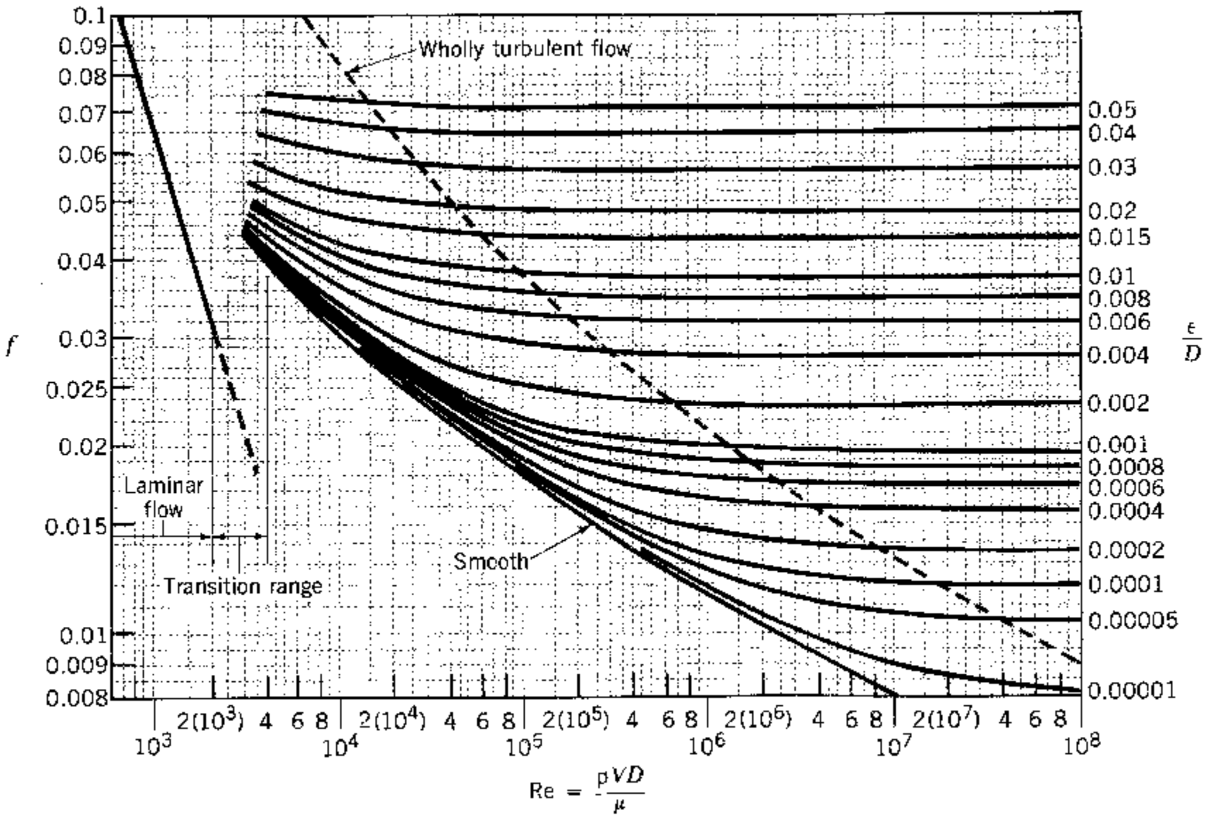


Fig A-1

PIPE ROUGHNESS

Pipe Type	**Roughness (ft)**
Glass	Smooth
Drawn tubing	0.000005
Asphalted cast iron	0.0004
Galvanized iron	0.0005
Cast iron	0.00085
Concrete	0.001 to 0.01
Riveted steel	0.003 to 0.03

Table T-1

LOSS COEFFICIENTS FOR FITTINGS AND VALVES

Fittings and Valves	K factor
Std 45 degree elbow	0.35
Std 90 degree elbow	0.75
Long radius 90 degree elbow	0.45
Coupling	0.04
Union	0.04
Gate valve open	0.20
¾ open	0.9
½ open	4.5
¼ open	24.0
Globe valve open	6.4
½ open	9.5
Tee (along run, line flow)	0.4
Tee (branch flow)	1.5

Table T-2

WATER PHYSICAL PROPERTIES

Temperature F	Density ρ (lbm/ft^3)	Dynamic Viscosity μ (lbf-sec/ft^2)*10^{-5}	Kinematic Viscosity v (ft^2/sec)*10^{-5}
32	62.42	3.75	1.93
40	62.43	3.22	1.66
50	62.41	2.74	1.41
60	62.37	2.36	1.22
70	62.3	2.02	1.04
80	62.22	1.81	0.935
90	62.12	1.58	0.817
100	62.00	1.44	0.749
110	61.87	1.28	0.665
120	61.71	1.18	0.614
130	61.55	1.07	0.559
140	61.38	0.981	0.514
150	61.20	0.906	0.475
160	61.00	0.830	0.437

Table T-3